International Studies Program

Working Paper 04-15
November 2004

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Fiscal Capacity Equalization and Economic Efficiency

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1. Introduction

Central governments in many countries including Canada, Germany, Switzerland, Japan, India, the UK and Australia use fiscal equalization models when distributing grants to sub-national governments. The models used vary somewhat in their precise structure but they do share general features. Some, for example the Australian model, estimate the revenue and expenditure needs of sub-national governments in generating the distribution of grant funds while others, such as Canada, estimate only the revenue needs of regions.

Cost disabilities may also be incorporated into schemes that take account of expenditure needs. The logic here is that high cost regions need more grant funds than a lower cost region simply to provide the same level of service. The additional funds are required in order to compensate for the relatively higher costs. An instance here is Switzerland where differences in the cost of providing services in mountainous areas are taken into account in calculating expenditure needs for equalisation - one disability proxy measures the relative significance of agricultural land above 800 metres. The Swiss also have proxies for population density in their allocation model (based on the idea that it is relatively more expensive to provide services to a geographically dispersed population). In Japan too, cost disabilities are included in the scheme that allocates equalisation grants to local governments. The types of disabilities accounted for include population density, population growth, climate, area and geography, degree of urbanisation, and industrial diversification. The UK also takes account of cost disabilities when allocating grants from the national government to local jurisdictions. These disabilities are constructed using a cross-section regression analysis.

Equalisation in practice is almost always motivated by equity concerns with the basic idea being to ensure equality of access to public services regardless of where a citizen lives. This is usually attempted by designing a system that attempts to equalise "fiscal capacities" across regions. The idea is illustrated from the following official statement of the aims of Australian equalisation:

> "*States should receive funding from the Commonwealth such that, if each made the same effort to raise revenue from its own tax bases and operated at the same level of efficiency, each would have the* capacity *to provide services at the same standard" (Commonwealth Budget Paper No. 3)*[1].

By equalising fiscal capacities through inter-State transfers it is thought that citizens of a federation with the same preferences and incomes can enjoy the same standard of State-provided public services with identical tax burdens regardless of where they live. A federation with equalised State fiscal capacities is one that, in principle, replicates the equity of a unitary system while at the same time provides the benefits of decentralisation, namely, the ability to have different packages of local public goods and taxes in accordance with local preferences.

Fiscal capacity equalisation results in inter-regional income transfers. Such transfers can be quite substantial, as the following Table shows for Australia. The Table shows in Column 1 how the pool of grant funds directed to the States is allocated using fiscal capacity equalisation. Column 2 shows how the funds would be distributed on a simple equal per capita basis, while the last column shows the difference between the two allocation methods. One can see that New South Wales, Victoria and Western Australia are "losers' while the remaining States and Territories are "winners".

Table 1: Equalisation, Australia, 2002-03

	Distribution using CGC model		**Equal per capita distribution**		
	$m (1)	**%**	**$m (2)**	**%**	**Difference (1) - (2)**
New South Wales	7723.9	29.0	8967.4	33.6	-1243.6
Victoria	5552.9	20.8	6650.7	24.9	-1097.8
Queensland	5236.9	19.6	5029.5	18.9	207.4
Western Australia	2447.8	9.2	2633.2	9.9	-185.3
South Australia	2641.6	9.9	2045.0	7.7	596.7
Tasmania	1118.6	4.2	637.8	2.4	480.8
ACT	519.6	1.9	429.3	1.6	90.3
NT	1423.6	5.4	272.1	1.0	1151.5
Total (Pool)	26664.9	100.0	26664.9	100.0	0.0

Source: Commonwealth Budget Paper No. 3, "Federal Financial Relations, 2002-03".

[1] The "Commonwealth" here refers to the Australian national government.

Economic efficiency arguments for such inter-regional transfers have appeared in the fiscal federalism literature. One, the 'efficiency in migration case', argues that local public goods create region-specific fiscal externalities, and that fixed factors of production create region-specific economic rents. The location decisions of mobile capital, and labour, are affected by these externalities and rents. The result is that in equilibrium these factors of production will be located inefficiently across regions. In short, 'too many' mobile factors locate in regions relatively rich in fiscal externalities and rents (eg. resource rich regions). It is argued that there is an optimal inter-State transfer that corrects for the distorting effects of externalities and rents, and establishes an optimal spatial distribution of mobile factors. Of course, the existence of an optimal transfer does not justify any of the schemes that we see in operation since none actually implement the optimal transfer[2].

Fiscal capacity equalisation has been criticised on efficiency grounds. Swan and Garvey (1991) attempt to show that it induces strategic behaviour by regions, or gaming against the distribution formula. The result is sub-optimal provision of local public goods by regions. It has also been argued that equalisation creates 'transfer dependency' (an issue for the Atlantic Provinces in Canada[3]) making regions reluctant to pursue economic development objectives. Equalisation may also slow down economic adjustments that would otherwise take place to facilitate inter-State income convergence, such as changes in relative wages between regions, or migration from low to high-income regions

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Therefore, though often justified on equity grounds by policy-makers, equalisation and inter-State transfers have been supported and criticised on economic efficiency grounds. There seems to be no consensus on the efficiency debate, with conclusions depending on particular lines of argument taken, assumptions made and modelling structures adopted.

This paper attempts to provide some cohesion to the efficiency debate. It does this by bringing together, within one model, the major efficiency rationale for inter-regional transfers - the presence of regional externalities and rents - together with one of the major efficiency arguments against equalisation: the potential for strategic behaviour by regions and sub-optimal provision of local public goods. The advantage of this approach is that it enables us to examine exactly how equalisation affects efficiency in a world in which regions act strategically, and where externalities and rents affect location decisions. We are also able to draw conclusions about the efficiency (and welfare) implications of equalisation in such a world.

The model adopted is novel in the sense that it integrates a real world fiscal equalisation scheme into a standard model of a decentralised (eg. federal) economy with optimising regional governments and factor mobility. Each region chooses the provision of a local public good to maximise the utility of a representative resident, while taking account of the migration responses to its decisions (regions are non-myopic with regard

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[2] Papers that analyse this idea include Boadway and Flatters (1982), Myers (1990) and Petchey and Shapiro (2000, 2002).

[3] See Courchene (1984).

to migration responses). The central government collects taxes in each region and distributes the revenue raised to the regions via an equalisation formula that estimates regional revenue and expenditure needs. The equalisation formula used is the Australian one. This is because Australian equalisation is, arguably, the most comprehensive in the world: it equalises for revenue and expenditure needs, and incorporates cost disabilities. As will be shown, many schemes used in other countries are special cases of the Australian approach.

The efficiency properties of a Nash equilibrium are then explored and the results are as follows. The first is that equalisation distorts State decisions over public policy directly, and indirectly, through migration responses to tax and spending decisions. The direct effect arises from the fact that changes in public policy affect the grant that is received by a State. This is because, at least in Australia's scheme, the standards used to assess whether a State has an expenditure and/or revenue need, are endogenous (they are functions of what the States actually do on average). Therefore, States, through their choice of public policy, are able to influence these standards, and hence the grant they receive. This gives them an incentive to distort their public policies away from what might otherwise be optimal. Whether the direct effect encourages States to over or under-produce local public goods, relative to optimal amounts, is shown to depend on the exact relationship between the expenditure and revenue needs in the equalisation scheme.

The indirect effect works through the migration responses to State policy choices. Specifically, if a State changes its public policy, and this affects its grant, then the latter will also affect mobile factor location choices. A State that is non-myopic will take account of these migration responses. This further distorts public policy choices (we also show that this migration response effect cannot be signed).

The second result relates to the efficiency of location choices made in a decentralised economy with equalisation. As noted, some have argued that in economies with free mobility, location choices are inefficient because of region-specific rents and fiscal externalities. This necessitates an optimal transfer. What we show is that in the equilibrium in our model there will be an inter-regional transfer, but it is not the one required for efficiency in the spatial allocation of mobile factors.

Therefore, we show that a decentralised economy with fiscal capacity equalisation experiences two kinds of inefficiency: one to do with sub-optimal provision of local public goods, and the other relating to an inefficient spatial allocation of mobile factors.

The paper outline is as follows. Section 2 develops the model of a decentralised economy with optimising sub-national governments, factor mobility and a fiscal equalisation system implemented by a central government. Section 3 examines public policy choices made by the sub-national governments, while Section 4 examines the efficiency and social welfare implications of those decisions in the presence of equalisation. Section 6 concludes.

2. Model

Suppose that we have a decentralised economy with N citizens who have identical incomes and preferences. For convenience we will think of this as a federal economy with i = 1,2 States. State i has n_i residents who each supply one unit of labour. The national population (labour supply) is therefore,

$$N = n_1 + n_2 . \qquad (1)$$

The production process in each State is simple. There are two inputs, the first, immobile and in fixed supply, can be thought of as land, fixed physical capital, or natural resources. We denote the supply of this factor in State i as L_i. The second factor is labour. Since each citizen supplies one unit of labour, n_i is State i's labour supply. As shown below, labour is perfectly mobile between States and its supply can vary from the perspective of each State. The two factors are combined using a production technology based on constant returns to scale to produce a numeraire good whose price is set at one. The value y_i of a State's production of the numeraire (the value of aggregate State output) is represented by the production function

$$y_i = f_i(n_i, L_i), \qquad i = 1,2 . \qquad (2)$$

Since the immobile factor is in fixed supply in each State, from now on we define the aggregate output of State i as $y_i = f_i(n_i)$ and suppose the following: $f_i^{'}(n_i) > 0$, $f_i^{''}(n_i) < 0$. Though States have the same production technologies we allow them to have different endowments of the fixed factor[4].

Competitive factor markets are also assumed implying that each person in a State receives a wage, w_i, equal to their marginal product. Since citizens of a State are identical, each receives the same wage, but because State specific supplies of land may differ, inter-State wage rates may not be the same. The residents of a State own equal portions of that State's fixed factor[5] and each receives an equal per capita share of the State's fixed factor income, or economic rent. Since we have assumed constant returns to scale, and hence that output is exhausted by factor payments, the income of a representative citizen in State i is the State's average product,

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[4] The aggregate State production function can be rationalised by assuming that there is a large number, for example, J_i, of perfectly competitive profit maximising firms in State i. All firms, denoted by the index j = $1 \ldots J_i$, are the same and hence have identical production functions, $y_i^j = h_i(\frac{n_i}{J_i}, \frac{L_i}{J_i})$. Aggregate production in State i is simply $J_i y_i^j$ and can be represented as in equation (2).

[5] This rules out the possibility that people may be 'absentee landlords' (ie. live in one State and own some of the fixed factor in another) and foreign ownership. The assumption also implies that as a person migrates from, say, State 1 to State 2, they immediately forfeit their right to fixed factor ownership in State 1, and gain a share of the fixed factor in State 2 upon entry to that State.

$$\frac{f_i(n_i)}{n_i}, \qquad i = 1, 2. \tag{3}$$

Part of the numeraire output in each State is transformed, by a State government, into a pure local public good denoted as q_i, with no inter-State spillovers, and the rest is consumed directly by State citizens. Per capita consumption of the numeraire is denoted as x_i. From now on we think of this as private good consumption. There is implicitly a transformation frontier defined between private and public good consumption that is assumed to be linear. The (constant) slope of the frontier is the marginal rate of transformation between the two goods that is equal to the marginal cost of x_i over the marginal cost of q_i. Under the assumption of perfect competition it is also equal to the price of the numeraire (one) over the price of the public good[6].

Each citizen has a quasi-concave, continuous and differentiable utility function,

$$u(x_i, q_i), \qquad i = 1,2. \tag{4}$$

As noted, citizens are also assumed to be perfectly mobile across States so that in equilibrium,

$$u(x_1, q_1) = u(x_2, q_2). \qquad \textit{(Equal utility condition)} \tag{5}$$

The equal utility condition can be thought of as a social welfare function; $W = u_1 = u_2$. Since citizens of a State receive location-specific fixed factor rents, and because of the presence of local public goods, which generate fiscal externalities, in this model labour will, in general, be allocated inefficiently between States. This is a well-known feature of federalism models with the underlying regional structure developed here.

2.1 Fiscal Capacity Equalisation

In practice, the size of the pool of funds to be distributed among the States is determined by tax and spending assignment between the national and sub-national governments, which commonly leads to a fiscal gap (an excess of revenue relative to expenditure) at the federal level. This gap is then distributed back to the States on the basis of various distribution formulas. We abstract from the complexities of how the pool is created and concentrate on the gaming behaviour of States over the distribution of the pool. It is the efficiency effects of the distribution of the pool, rather than any distortions related to the creation of the pool, which are of primary focus here.

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[6] Defining p_i as the price of the public good, the marginal rate of transformation is simply $1/p_i$. With a linear frontier this ratio is unchanged as we adopt different combinations of x_i and the public good. Therefore, p_i is treated as a parameter. Since it can differ across States, the ratio $1/p_i$ can also vary across States. Thus, generally States will have differently sloped frontiers.

Therefore, it is supposed that some pool G is created by a federal government using a per capita lump sum tax on citizens, denoted as s. In addition, we do not model central government provision of public goods (national public goods). Rather, the only role given to the central government is one of creating a revenue pool that is then distributed to the States using an equalisation methodology. This is clearly a major abstraction and simplification of central government behaviour, but again, it is one that allows us to focus on the issue at hand: the distortions created by gaming over the allocation of a given pool.

For simplicity we also assume that s is given[7]. It represents a quantity of the numeraire that is surrendered by a citizen to the national government. Since the numeraire produced in each State is the same the quantity collected by the national government can be aggregated to create a single 'pool' of the numeraire, denoted as G = sN. Since s and N are fixed, G is a parameter.

2.2 The State - Specific Grant

As noted, we have chosen to model equalisation using the Australian approach. The grant pool in Australia is allocated between the States using the Commonwealth Grants Commission (CGC) equalisation formula[8]. We integrate the key components of this formula into the federal model, and in doing so, abstract from the inessential parts. Though the formula applies in a federation of multiple States, we also suppose there are only two States, i = 1,2, consistent with our model. The CGC's formula defines the per capita grant, g_i, to State i as:

$$g_i = \frac{G}{N} + \frac{E}{N}(\gamma_i - 1) + \frac{T}{N}(1 - \rho_i) + c, \qquad i = 1,2. \qquad (6)$$

G and N are the total grant pool and national population (as previously defined) and so G/N = s is the per capita amount of funds available for distribution to the States.

The variable E is defined by the CGC as total expenditure by the States on the services included in the model (for example, education, health, transport, welfare). Here, there is only one service, a local public good, so that the following holds: $E = p_1 q_1 + p_2 q_2$. Therefore, E/N is the per capita average expenditure on the local public good across both States. The CGC calls this 'standard expenditure'. As will be shown below, it is the expenditure that is used as a benchmark to assess a State's 'expenditure need'.

The variable T is defined by the CGC as the total revenue raised by all States to fund their public expenditures (own-source revenue). This is equal to total expenditure by all the States, less what they receive as grants from the federal government, G.

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[7] The determination of s could be made endogenous by explicitly modelling national government optimising behaviour. If States took account of the impact of their policy decisions on s, then additional distortions, not directly related to equalization, would be introduced. We abstract from these considerations by supposing that States treat s as given.

[8] See CGC (1999).

Therefore, own-source revenue can be defined as T = E – G, or alternatively, $T = p_1q_1 + p_2q_2 - G$. Furthermore, T/N is the per capita average own-source revenue raised by all States, known as ‘standard’ revenue in CGC terminology. Again, this name is given to the term T/N because, as will be seen, it is the benchmark used to assess whether a State has a ‘revenue need.’

2.3 Cost and Revenue Disabilities

Another part of (6) is the cost disability, γ_i. It captures the cost of providing each service in State i, relative to the average cost for all States. The CGC calculates a cost disability for each service provided by the States. The calculations are complex and since we have only one service we have only one cost disability for each State (for the local public good).

A State may have a cost disability in the provision of a particular service for a variety of reasons. For example, it may have a geographically dispersed population and have to provide schools in remote locations. This means that a unit of education service may have a higher cost than the average across all States. Other factors contributing to cost disabilities include the age/sex profile of the population, ethnicity and the presence of groups with special health/educational requirements and economies of scale. Australian equalisation is unique in the sense that it puts a great deal of effort into estimating such cost disabilities and then allowing them to determine the distribution of the grant pool, G.

We adopt a simplified definition of a cost disability that, of necessity, abstracts from this complexity, but captures the essence of the idea. Namely, we define the cost disability for State i as

$$\gamma_i = \frac{2p_i}{p_1 + p_2}, \qquad i = 1,2. \qquad (7)$$

So defined, if $\gamma_i > 1$, then State i is a relatively high cost provider of the public good (has a cost disability) and if $\gamma_i < 1$, it is a relatively low cost provider. Thus, the cost disability variable is normalised around the number one. The prices of the public good are exogenous so the disability is treated as exogenously given by the States.

The CGC also estimates a State specific ‘revenue disability’ for each State tax base. In Australia’s case, such disabilities are estimated for all the State taxes, including payroll tax, the major State tax, and mineral royalties. Again, we abstract from this complexity and suppose that a State has only one revenue disability, ρ_i, which is greater than one if the State has a relatively strong tax base, and less than one if the State has a relatively weak tax base. As with the cost disability, we assume that States take the revenue disability as determined exogenously by the CGC.

2.4 Expenditure and Revenue Needs

The term $(E/N)(\gamma_i - 1)$ in equation (6) is the expenditure need of State i. If we multiply E/N through the brackets we can see that the need has two parts. The first, $(E/N)\cdot\gamma_i$, is the standardised expenditure of State i. This is the expenditure that State i would have to undertake, taking account of its cost disability, to achieve the per capita standard, E/N. State i's standardised expenditure is greater than or less than the standard, depending on the magnitude of its cost disability. The second term, E/N, is just the standard expenditure of all States. Thus, the expenditure need of the State is equal to its standardised expenditure less standard expenditure. If the State's cost disability is greater than one, the State has a positive expenditure need. Otherwise, it is negative.

Similarly, $(T/N)(1-\rho_i)$ is the revenue need of State i. Multiplying through the brackets one can see that it also has two parts. The first is just T/N, or standard own-source revenue. The second term, $(T/N)\cdot\rho_i$, is the standardised own-source revenue of State i. This is the revenue that State i would raise if it applied the average tax effort to its own tax base. If the State's revenue disability is greater than one, then its standardised revenue will be higher than the standard, and its revenue need will be negative. Alternatively, if the State's revenue disability is less than one, its standardised revenue will be less than the standard, and the State will have a positive revenue need.

Thus, under Australian equalisation a State receives an equal per capita share of the pool, G/N, adjusted by the expenditure and revenue need terms. A State will receive more than its equal per capita share of the grant pool if the sum of its needs is positive; and less than its equal per capita share if the sum of its needs is negative. Finally, if the expenditure and revenue needs cancel each other exactly, the State will simply receive its equal per capita share, G/N.

Note also here that, due to the differences in their definition, the disabilities are applied differently in equation (6). Namely, for the expenditure disability standard expenditure is multiplied by $(\gamma_i - 1)$ where $\gamma_i > 1$ implies that the State has relatively high costs, while for the revenue disability standard revenue is multiplied by $(1-\rho_i)$ where $\rho_i > 1$ denotes a State with a relatively rich tax base.

2.5 Balanced Grant Pool Condition

As shown above, the per capita grant to a State is estimated on the basis of its expenditure and revenue needs. Hence, there is no reason, a priori, for the sum of the aggregate grants across all States to exactly exhaust the available grant pool, G. Therefore, one must introduce an adjustment to the per capita needs-based grant estimated for each State to ensure that a 'balanced grant pool condition' is satisfied, i.e. that the sum of the aggregate grants exactly exhausts the pool. The last term, c, in (6) is included to do just that.

One can incorporate this adjustment explicitly into the modeling, but there are two problems with doing this. One is that it is not clear to us exactly how the CGC does this adjustment, and second, if some balanced grant pool condition is explicitly incorporated (eg. a simple condition that the sum of the aggregate State specific grants must equal G) the mathematics of the State's optimisation problem becomes overly complex and obscures the key results (without adding anything of economic interest).

Therefore, we abstract from the complexity of such an approach by assuming an additive form for the adjustment. Thus, the additive term c in (6) is assumed to be the adjustment made by the CGC to each State's per capita grant estimated on the basis of needs.

What is the formula for the required adjustment, and why do we choose to make it the same in each State? The following procedure answers both questions. We start with the balanced grant pool condition:

$$n_1 g_1 + n_2 g_2 = G \qquad (8)$$

Substituting the State specific grants given by (6) into (8), we obtain an equation for the calculation of c, which will ensure that (8) is satisfied. This is

$$c = \frac{G}{N} - \frac{E}{N}(n_1\gamma_1 + n_2\gamma_2) + \frac{T}{N}(n_1\rho_1 + n_2\rho_2)\,. \qquad (9)$$

From this it is evident that the adjustment to the grant formula needs to be identical across States. If the adjustment was State specific, we would have (in our two State example) only one equation, (8), to identify two unknowns (two State specific adjustments). Therefore c has to be the same across States, and it is given by (9).

The fact that the CGC is assumed to set c so that (8) is satisfied, also implies that the following must hold

$$g_1 n_1 + g_2 n_2 = s(n_1 + n_2) = sN = G\,, \qquad (i = 1,2). \qquad (10)$$

Equation (9) gives rise to an interesting issue of whether States are aware of how c is calculated. In practice, as mentioned earlier, the precise methodology for the CGC's calculation of the adjustment is not clear, and one could assume that for this reason States view c as exogenously given and do not take it into account when making their public policy decisions. However, one can also expect States to be aware that, as is obvious from (9), the balanced budget adjustment is a function of their public good provision choices. It is, therefore, reasonable to expect that States would understand that some adjustment is needed to meet the balanced grant pool condition. For this reason we incorporate c explicitly into the model thus introducing additional strategic bahaviour and potential distortions to State policy-making.

2.6 Summary

Each endogenous variable in the model is a function of the exogenous variables and parameters. For later discussion, it is useful to explore this in more detail for one of the endogenous variables, for example the grant to State i. In this regard, from (6) one can define the per capita grant to a State as

$$g_i(F, P, CGC, S) \tag{11}$$

where $F = [s\ N]$ is a vector of variables determined by the federal government, $P = [p_1 p_2]$ is a vector of the local public good prices, $CGC = [\gamma_i\ \rho_i\ c]$ is a vector of variables determined by the CGC and $S = [q_1 q_2]$ is the strategy set of the two States. Within F, the variable s is determined by the federal government. The total federal population N is determined by things such as the birth and death rate, but also by international migration and hence, to some extent, the population policy of the federal government. Within the vector CGC, the variables γ_i, ρ_i, c are all determined by the CGC, while the public good provision levels within S are determined by the States.

As discussed below, we assume that each State perceives s, N, public good prices and the CGC variables (except the adjustment term c) to be exogenously given. This is reasonable since in practice the States have no impact on s and only a marginal impact on the CGC variables. It is true that the States rent seek over the cost and revenue disabilities, but in reality, this meets with limited success. As also discussed below, we assume that each State adopts Nash conjectures with regard to the level of provision of the public good in the other State. For example, when optimising, State 1 perceives q_2 to be given, and similarly for State 2. Thus, each State perceives its grant to be a function of its own public good provision. The adjustment variable c is also among the variables contained in the vector CGC. Given the previous discussion, we know that States also view c as a function of their joint policy choices.

The general function (11), which links State policies and equalisation variables to the State specific grant, will hold in any federation with equalisation, though the specific variables to be included will vary depending on the particular structure of the equalisation formula used. Thus, one can think of (11) as a general function defining the State specific grant, and equation (6) as the specific function for the Australian model.

3. State Policy Choices

Taking into account private good consumption, provision of the public good, the national government tax and the equalisation grant, the aggregate budget constraint in State i is:

$$n_i x_i + p_i q_i + n_i s = f_i(n_i) + n_i g_i\,, \tag{12}$$

Thus, total expenditure on the private and public good, and the revenue paid to the central government, must equal total State output plus the revenue from the equalisation grant.

Suppose that the government of each State is benevolent and perfectly represents the preferences of its citizens. The implication is that State and citizen interests are synonymous: a State will choose its level of provision of the public good to maximise per capita utility within its jurisdiction (recall that all residents within a State have the same income and preferences). In making its choice, a State is assumed to take account of the equal utility condition and the equation defining the total supply of labour. Thus, States are non-myopic with regard to the migration effects of their public policy choices. It is also assumed that States take account of the grant response to any changes in public good provision, through the equalisation formula. This implies that each State's choice of public good provision will affect the welfare of citizens in the neighbouring State, both through the migration condition and the equalisation formula.

Such policy interdependence means that States can act strategically. One can, therefore, think of the problem as a two-player simultaneous move game in continuous pure strategies, in which the States are players and the payoffs are the per capita utilities in each State. The strategy set for the game, defined previously, is $S = (q_1, q_2)$. Nash conjectures are assumed so each State chooses its public good provision taking provision in the other State as given. However, it is supposed that States have sufficient foresight to take account of the impact of their choice on migration and the equalisation grant. Given this, each State will choose its provision of the public good to maximise within-State per capita utility subject to the State-specific budget constraint, the national labour supply condition, the equal utility condition and the equalisation formulas.

3.1 State Optimisation

For convenience, from now on we conduct the analysis from the perspective of State 1. Rewriting the aggregate budget constraint for State 1 in terms of per capita private good consumption and substituting the result into the per capita utility function, the problem of State 1 can then be written as follows[9]:

$$\underset{(q_1)}{\text{Max}} \; u\left(\frac{f_1(n_1) - p_1 q_1}{n_1} - s + g_1, q_1\right) \tag{13}$$

[9] We also assume here that States are unable to make lump-sum inter-State transfers, as in Myers (1990). This seems reasonable for a federation where States have never been observed to use such transfers. However, as will be discussed later, equalization does lead to an inter-State transfer, and this transfer is a function of joint-State policies. Hence, States in our model do indirectly choose the inter-State transfer via the equalization process.

Subject to:

Migration and Labour Supply Constraints

(i) $u\left(\frac{f_1(n_1)-p_1q_1}{n_1}-s+g_1,q_1\right)=u\left(\frac{f_2(n_2)-p_2q_2}{n_2}-s+g_2,q_2\right)$

(ii) $n_1+n_2=N$

Fiscal Capacity Equalisation Constraints

(iii) $g_1=\frac{G}{N}+\frac{E}{N}(\gamma_1-1)+\frac{T}{N}(1-\rho_1)+c$

(iv) $g_2=\frac{G}{N}+\frac{E}{N}(\gamma_2-1)+\frac{T}{N}(1-\rho_2)+c$

(v) $c=\frac{G}{N}-\frac{E}{N}(n_1\gamma_1+n_2\gamma_2)+\frac{T}{N}(n_1\rho_1+n_2\rho_2)$

(vi) $E=p_1q_1+p_2q_2$.

(vii) $T=E-G$.

From State 1's perspective the exogenous variables are s, N, p_1, p_2, G (defined as $G=sN$), γ_1,γ_2 (defined by (7)) and $\rho_{1,}\,\rho_2$. These are the variables determined by the federal government and the CGC, and perceived to be exogenous by the States. With Nash conjectures, State 1 will also treat q_2 as given. The endogenous variables are, therefore, q_1, n_1, n_2, g_1, g_2, E, T and c, which is an endogenous function of both exogenous and endogenous variables. The constraint set has seven equations. There are 8 unknowns and hence one free dimension to maximise over. Also, we do not use G = sN in constraints (iii) and (iv) because we want these constraints to reflect the way that the CGC formulates its model. Of course, if we do use G = sN in constraints (iii) and (iv) then G/N simply becomes s, the per capita tax levied by the federal government[10]. State 2 solves an analogous problem with identical exogenous variables and with Nash conjectures it takes q_1 as given when it chooses q_2.

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[10] A State also perceives its population size to be a function of its own policy choice. To see this, note that for given $q_1, q_2, p_1, p_2, s, g_1$ and g_2, constraints (i) and (ii) determine n_1 and n_2. We know that g_1 and g_2 are functions of s, N, public good prices, CGC variables and joint State policies. Therefore, for State 1, $n_1(F, P, CGC, S)$ where F, P, CGC and S are as previously defined. State 1, when optimising according to (13) with Nash conjectures, will perceive everything to be fixed, except its own level of provision of the public good. Hence, it perceives its population to be a function of its own policy choices.

3.2 Efficiency and Coincident Interests

The set up above implies that each State will seek to maximise utility for a representative State citizen, taking into account the migration, budget and equalisation constraints. Through the equal utility condition we know that $u_1 = u_2$, and because States recognise this, each will know that to maximise its own per capita utility, it will have to maximise equal per capital utility, and hence utility in the neighbouring State. Thus, when choosing their policies each State maximises the social welfare function $W = u_1 = u_2$. States have a coincidence of interests, and even though they participate in a non-cooperative game, they each seek efficient outcomes in the sense that they attempt to make $W = u_1 = u_2$ as large as possible, given whatever exogenous constraints they face. Perfect mobility means that States have an interest in pursuing efficient outcomes since this makes W (national welfare), and hence the utility of their own residents, as big as is feasible.

3.3 Local Public Good Provision

The necessary condition for public good provision is found by differentiating the objective function in (13) with respect to q_1, for given q_2 under Nash conjectures, and fixed values of s, N, the CGC variables (except adjustment factor c) and public good prices. This yields

$$n_1 MRS^1_{xq} + b_1 \cdot \frac{\partial n_1}{\partial q_1} + n_1 \frac{\partial g_1}{\partial q_1} = p_1 , \qquad (14)$$

where $MRS^1_{xq} = u_{q_1} / u_{x_1}$ is the marginal rate of substitution between the local public good and the private good in State 1. The remaining parts of (14) are explained below. The term b_1 is the net benefit of an additional migrant in State 1 and is defined as

$$b_1 = (w_1 - x_1) - (s - g_1) , \qquad (15)$$

where $(w_1 - x_1)$ is the difference between a migrant's marginal product (wage) and their per capita consumption, and $(s - g_1)$ is the difference between the federal tax they pay (output foregone by State 1) and the per capita equalisation grant the migrant attracts to the State. The net benefit of an extra migrant for a State is more complex than in traditional federal models because we have to account for the grant and equalisation consequences of migration into a State[11].

[11] In standard models where States are not linked through an equalization model, the net benefit of a migrant is just $b_1 = (w_1 - x_1)$.

3.4 Own - Grant Responses

The term $\frac{\partial g_1}{\partial q_1}$ in (14) is the change in State 1's equalisation grant in response to a small increase in the State's public good provision (the own-grant response). An expression for this response can be found by differentiating constraint (iii) with respect to q_1, for given q_2, and fixed values of the exogenous variables. This yields[12]:

$$\frac{\partial g_1}{\partial q_1} = \frac{p_1}{N}(\gamma_1 - 1) + \frac{p_1}{N}(1 - \rho_1) + \frac{\partial c}{\partial q_1}. \qquad (16)$$

The first term on the right side, $(p_1 / N)(\gamma_1 - 1)$, is the change in the expenditure need of State 1 resulting from a small change in its provision of the local public good. The expenditure need changes because a variation in the provision of the local public good changes standard expenditure, which is in turn used by the CGC in its per capita grant formula to determine the standardised expenditure of each State, and hence the State specific expenditure needs. Similarly, the term $(p_1 / N)(1 - \rho_1)$ is the change in the revenue need of State 1 resulting from a small change in its provision of the local public good. Again, the revenue need changes because, as discussed previously, standard revenue, T/N, is a function of the expenditure choices of the States. The third term, $\partial c / \partial q_1$, is the change in the balanced budget condition adjustment c.

The total change in the grant is just the sum of the changes in the expenditure and revenue needs, and the balanced budget adjustment. The sign and magnitude of the changes in the two types of need, and the adjustment, will determine the sign and magnitude of the own-grant response term. The sum of these changes can also be expressed simply as $(p_1 / N)(\gamma_1 - \rho_1) + \partial c / \partial q_1$.

If we were to assume that States have no foresight as to how the CGC makes the balanced budget adjustment (i.e. $\partial c / \partial q_1 = 0$), we could have more insight into the implications of (16). In that case, if the cost disability for State 1 were greater than the revenue disability, the own-grant response would be positive. Thus, the direct effect of fiscal equalisation would be to increase the marginal benefit of q_1 because more q_1 attracts more grant. If the cost disability were less than the revenue disability, the own-grant response term would be negative: more q_1 reduces the grant and hence the marginal benefit of the public good.

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[12] It is also possible to derive an explicit expression for $\frac{\partial c}{\partial q_1}$. It turns out to be

$$\frac{\partial c}{\partial q_1} = \frac{p_1}{N}\left[n_1(\rho_1 - \gamma_1) + n_2(\rho_2 - \gamma_2)\right] + \frac{p_1 q_1}{N}\left[\frac{\partial n_1}{\partial q_1}(\rho_1 - \gamma_1) + \frac{\partial n_2}{\partial q_1}(\rho_2 - \gamma_2)\right].$$ However, there is no determinate way to sign this expression, and due to its complexity, it has little intuitive meaning. Therefore, in the text below we refrain from writing out the explicit expression.

More generally, and especially with the absence of the $\partial c/\partial q_1 = 0$ assumption, the sign of the own-grant response is indeterminate, and depends on the relative magnitude of the cost and revenue disabilities, and the change in the balanced budget adjustment.

3.5 Own - Migration Response

The term $\frac{\partial n_1}{\partial q_1}$ in (14) is the change in State 1's population resulting from a small increase in its provision of the local public good (an own-migration response). An expression for this can be found by differentiating constraints (i) and (ii) with respect to public good provision in State 1 yielding, in matrix form Ax = d:

$$\begin{pmatrix} b_1 \cdot \frac{u_{x_1}}{n_1} & -b_2 \cdot \frac{u_{x_2}}{n_2} \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \frac{\partial n_1}{\partial q_1} \\ \frac{\partial n_2}{\partial q_1} \end{pmatrix} = \begin{pmatrix} u_{x_1}\frac{p_1}{n_1} - u_{q_1} - u_{x_1}\frac{\partial g_1}{\partial q_1} + u_{x_2}\frac{\partial g_2}{\partial q_1} \\ 0 \end{pmatrix} \quad (17)$$

where b_1 is as described previously and $b_2 = (w_2 - x_2) - (s - g_2)$ is the net benefit of an additional migrant in State 2.

The d vector includes the own-grant response for State 1, and the response of State 2's grant to changes in State 1's public good provision. This is because, when State 1 changes its provision of the public good, it changes its fiscal equalisation grant, as just discussed, but also, the grant received by its neighbour. This, in turn, influences people's location choices (people move in response to a change in the distribution of the grant pool G). Since States are non-myopic (by assumption) with respect to the migration effects of their decisions, and hence take the equal utility condition into account, they must also account for the migration consequences of changes in the fiscal equalisation grants as they change public good provision.

We already have an expression for the own-grant response term for State 1. A similar expression for the response of State 2's grant to changes in State 1's public good provision is found from constraint (iv) to be:

$$\frac{\partial g_2}{\partial q_1} = \frac{p_1}{N}(\gamma_2 - \rho_2) + \frac{\partial c}{\partial q_1} \quad (18)$$

Using (17), the own-migration response in State 1 is:

$$\frac{\partial n_1}{\partial q_1} = \frac{1}{D}\left(\frac{u_{x_1}}{n_1}p_1 - u_{q_1} - u_{x_1}\frac{\partial g_1}{\partial q_1} + u_{x_2}\frac{\partial g_2}{\partial q_1}\right) \quad (19)$$

where $D = b_1 \frac{u_{x_1}}{n_1} + b_2 \frac{u_{x_2}}{n_2}$ is the determinant of the A matrix in (17). The sign of the response is indeterminate[13].

3.6 Equilibrium

The own-migration and own-grant expressions can be substituted into the necessary condition for State 1 to derive a single expression showing public good provision in State 1 as a function of given values of the exogenous variables, parameters and public good provision in State 2 (Nash conjectures). This expression is the best reply function for State 1.

Solving an analogous problem for State 2 also yields the necessary condition for public good provision in State 2 as

$$n_2 MRS^2_{xq} + b_2 \frac{\partial n_2}{\partial q_2} + n_2 \frac{\partial g_2}{\partial q_2} = p_2 \qquad (20)$$

It is straightforward to derive the net benefit, own-migration and own-grant responses for State 2, and substitute these responses into (20) to obtain a single equation for public good provision in State 2 as a function of given values of the exogenous variables, parameters and public good provision in State 1 (Nash conjectures). This is the best reply function for State 2.

A Nash equilibrium, assuming that one exists, is simply the strategy set, $S^* = (q_1^*, q_2^*)$ that solves the two best reply functions simultaneously.

4. Welfare

As noted the States have a coincidence of interests and choose their policies to maximise $W = u_1 = u_2$. We now examine how the Nash equilibrium characterised above performs in welfare terms. Two approaches are used to think about this issue. The first looks at whether public good provision is efficient in each State, while the second examines whether the mobile labour force is allocated optimally between States in equilibrium.

3.6 Under or Over Provision?

Efficiency in the provision of the local public goods requires that the Samuelson condition, $n_1 MRS^1_{xq} = p_1$ be satisfied (similarly for State 2). Clearly, this is not so in our model because of the presence of the migration and grant responses. Hence, the local

[13] In the absence of equalization, substitution of the own-migration response into the public good necessary condition yields the Samuelson condition (see Annex A). This is the result of Boadway (1982).

public goods will be over or under provided relative to levels of provision consistent with the Samuelson condition. However, because of the indeterminacy of the sign of these responses we are unable to draw any general conclusion about whether equalisation leads to over or under provision of local public goods.

There is one case where it is possible to be more certain. If States are myopic with respect to migration responses, and also perceive $\partial c / \partial q_1 = 0$, then the migration and adjustment response terms drop out of the public good necessary condition for State 1. Using (16), the necessary condition (14) then becomes:

$$n_1 MRS^1_{xq} + n_1\left(\frac{p_1}{N}(\gamma_1 - \rho_1)\right) = p_1 \tag{21}$$

If the cost disability is greater than the revenue disability parameter, a case that is more likely in a high cost and low income State, then the own-grant response to more public good provision is positive. In this case, equalisation increases the marginal benefit of extra units of the public good (more public good attracts more grant) and raises public good provision above optimal levels. Similarly, if the cost disability is lower than the revenue disability parameter, a case that might apply in a relatively rich, low cost State, then the own-grant response is negative. Equalisation reduces the marginal benefit of extra units of the public good (more public good reduces the State's grant) and lowers public good provision below optimal levels.

Thus, in a federation where States act myopically with regard to the migration responses to their public good provision, and perceive that $\partial c / \partial q_i = 0$, fiscal capacity equalisation raises public good provision in high cost and low income States above optimal levels, and lowers public good provision below optimal amounts in States with relatively low costs and high income.

3.7 Spatial Distribution of Labour

The other interesting question from an efficiency perspective is whether the mobile population will be distributed optimally across States in equilibrium. This issue can be assessed by examining the first-best Pareto optimal outcome in a federation that has the same regional structure as our model. The Pareto optimal outcome is characterised by supposing that it is governed by a benevolent central planner who chooses the private and public good provision in either of the regions to maximise per capita utility in that region. At the same time, the utility of a representative citizen in the other State is held fixed at some predetermined level. The central planner also takes into account the national feasibility and labour supply constraints[14].

The solution is presented in Annex B where it is shown that there are two necessary conditions for a Pareto optimum. One is that the Samuelson rule must hold in

[14] For example, see Flatters, Henderson, Vernon and Mieszkowski (1974) and Myers (1990) who solve analogous central planner problems to explore Pareto optimality.

each State (as shown above, this is not the case in our model). The second is that in equilibrium the mobile population must be distributed between States such that the following 'equating at the margin rule' is satisfied,

$$(w_1 - x_1) = (w_2 - x_2)\,, \qquad (22)$$

It is well known that this condition will in general not be satisfied because of the presence of fiscal externalities and location specific economic rents which distort migration decisions. There is an efficient transfer from State 1 to State 2 that corrects for this distortion and establishes an optimal distribution of the mobile population consistent with (22). This transfer is found by solving for t from $(w_1 - x_1 - t/n_1) = (w_2 - x_2 + t/n_2)$ to yield[15]:

$$t^{opt} = \frac{n_1 n_2}{N}\cdot\big((w_1 - x_1) - (w_2 - x_2)\big) \qquad \textit{(Efficient Transfer)} \quad (23)$$

For the mobile population to be allocated efficiently across States in our model, the inter-State transfer that results from the equalisation process would have to be consistent with this efficient transfer. We know from (10) that $\sum_i g_i n_i = s\sum_i n_i$ must hold in equilibrium because the CGC is assumed to be setting the adjustment parameter in each State's per capita grant formula to ensure that the condition is satisfied. This implies that the equilibrium inter-State transfer under equalisation, denoted by t^E, is:

$$t^E = n_1(s - g_1) = n_2(s - g_2)\,. \qquad \textit{(Transfer with Equalisation)} \quad (24)$$

Since g_1, g_2, n_1 and n_2 are functions of State policies, for given values of the federal government variables, public good prices and CGC variables, t^E is a function of collective State policies. Thus, through the equalisation formula, the inter-State transfer is determined by the collective policies of the States. But there is no reason why the transfer under equalisation should be the same as the transfer required for efficiency: hence, $t^E \neq t^{opt}$ and the distribution of labour is inefficient.

One can conclude from this that even though the States want to maximise $W = u_1 = u_2$ the Nash equilibrium is Pareto dominated by the efficient central planner's solution for two reasons: first, equalisation distorts State perceptions of the marginal benefit from public good provision, and second, the inter-State transfer that results under equalisation is inconsistent with the efficient transfer.

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[15] See also Boadway and Flatters (1982). We can write the optimal transfer alternatively as a function of State specific rents and fiscal externalities (see Petchey (1993)).

5. Conclusion and Implications

We develop a model that is novel since it integrates a real world fiscal capacity equalisation scheme, namely, the Australian one, into a standard model of a decentralised (eg. federal) economy with optimising regional governments and factor mobility. This allows us to examine the efficiency of an equilibrium where States are linked together through fiscal capacity equalisation and where factors of production are mobile and make location choices while taking into account region-specific rents and externalities.

The key result is that the equilibrium with fiscal capacity equalisation is inefficient for two reasons. The first is that it distorts State decisions over public policy directly, and indirectly, through migration responses to tax and spending decisions. The direct effect arises from the fact that changes in public policy affect the grant that is received by a State. The indirect effect works through the migration responses to State policy choices. The second source of inefficiency is that the inter-State transfer that results under fiscal capacity equalisation is not consistent with the inter-State transfer required for economic efficiency. Thus, in the equilibrium with fiscal capacity equalisation the spatial allocation of mobile factors is inefficient.

What are the implications for the design of systems of fiscal equalisation? The first is that standards used in fiscal capacity equalisation models may need to be exogenously set. In the Australian model they are endogenous (see equation (6) where E - itself a function of State policies - is an argument in the grant function for State i), and this is the source of the inefficiency due to strategic behaviour. Such a finding raises the question of how one should set the standards, and where they should come from, if they are not be a reflection of what regions do on average. The second implication relates to the inter-State transfer under fiscal capacity equalisation. As we have shown, it yields an inefficient spatial allocation of mobile resources because it is determined by factors such as cost disabilities, and not fiscal externalities. So no matter what design changes are made to the equalisation formula, this source of inefficiency will remain.

Annex A: Substitution of the Migration Response

In the absence of equalisation, the local public good necessary condition is:

$$n_1 MRS^1_{x,q} + \frac{\partial n_1}{\partial q_1}(w_1 - x_1) = p_1 . \quad \text{A1}$$

The own-migration response is

$$\frac{\partial n_1}{\partial q_1} = \frac{1}{D}\left(\frac{u_{x_1}}{n_1} p_1 - u_{q_1}\right) \quad \text{A2}$$

where

$$D = \frac{u_{x_1}}{n_1}(w_1 - x_1) + \frac{u_{x_2}}{n_2}(w_2 - x_2) . \quad \text{A3}$$

Substituting A2 into A1 yields:

$$n_1 \frac{u_{q_1}}{u_{x_1}} + \frac{1}{D}\left(\frac{u_{x_1}}{n_1} p_1 - u_{q_1}\right)(w_1 - x_1) = p_1 \quad \text{A4}$$

Multiplying through by D gives:

$$n_1 \frac{u_{q_1}}{u_{x_1}} \cdot D + \left(\frac{u_{x_1}}{n_1} p_1 - u_{q_1}\right)(w_1 - x_1) = p_1 \cdot D \quad \text{A5}$$

Using A3 in A5, and then expanding and simplifying yields:

$$n_1 \frac{u_{q_1}}{u_{x_1}} = p_1 . \quad \text{A6}$$

Annex B: The Pareto Optimal Problem

The Pareto optimal (central planner's) problem is to

$$\underset{\substack{(x_1, x_2 \\ q_1, q_2 \\ n_1, n_1)}}{\text{Max}} \; u(x_1, q_1) \quad \text{B1}$$

subject to

(i) $(n_1 x_1 + n_2 x_2) + (p_1 q_1 + p_2 q_2) = f_1(n_1) + f_2(n_2)$

(ii) $u(x_2, q_2) = \bar{u}_2$

(iii) $N = n_1 + n_2$.

The Lagrangian is:

$$L = u(x_1, q_1) + \lambda_1\left(u(x_2, q_2) - \bar{u}_2\right) + \phi\left(\left(f(n_1) + f_2(n_2)\right) - (n_1x_1 + n_2x_2) - (p_1q_1 + p_2q_2)\right) + \mu\left(N - n_1 - n_2\right) \qquad \text{B2}$$

The first order conditions are:

$$x_1: \quad \frac{\partial L}{\partial x_1} = u_{x_1} - \phi n_1 = 0 \qquad \text{B3}$$

$$q_1: \quad \frac{\partial L}{\partial q_1} = u_{q_1} - \phi p_1 n_1 = 0 \qquad \text{B4}$$

$$x_2: \quad \frac{\partial L}{\partial x_2} = u_{x_2}\lambda_2 - \phi n_2 = 0 \qquad \text{B5}$$

$$q_2: \quad \frac{\partial L}{\partial q_2} = u_{q_2}(1+\lambda) - \phi p_2 n_2 = 0 \qquad \text{B6}$$

$$n_1: \quad \frac{\partial L}{\partial n_1} = \phi(w_1 - x_1) - \mu = 0 \qquad \text{B7}$$

$$n_2: \quad \frac{\partial L}{\partial n_2} = \phi(w_2 - x_2) - \mu = 0 \qquad \text{B8}$$

The term $(w_1 - x_1)$ in B7 is the net benefit of an additional worker in State 1. This consists of their wage, or marginal product, less their per capita consumption of the private good. Similarly, $(w_2 - x_2)$ in B8 is the net benefit from an additional worker in State 2. Using B3 and B4 together, and B5 and B6, yields the Samuelson conditions,

$$n_1 mrs^1_{xq} = p_1, \qquad n_2 mrs^2_{xq} = p_2. \qquad \text{B9}$$

The remaining equations for population imply that

$$(w_1 - x_1) = (w_2 - x_2) \qquad \text{B10}$$

Thus, Pareto optimality requires that workers be allocated across States so that the net marginal benefit from an extra worker is the same in each State (an ‘equating at the margin rule’), and that the Samuelson condition apply in each State.

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