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International Studies Program
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On Some Non-Equivalence Results of Ad Valorem Tax[*]

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Abstract:

This paper reviews some established non-equivalence results of an *ad valorem* tax, provides a general proof for those results, and presents some evidence illustrating that policy makers may be neglecting the subtlety of non-equivalence in establishing policies such as revenue maximizing tax rates.

JEL Classification Numbers: H0, H2

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1. Introduction

It is well understood and easy to see that the incidence of a *unit tax* is irrelevant to how the tax is legally imposed: the incidence is independent of whether the tax is imposed on the consumer or on the producer. See, for example, Musgrave (1959) and Rosen (2002). This result also has implications for the optimal tax rate: regardless of the objective (revenue maximizing, or welfare enhancing, for example), the same tax rate levied on producer or consumer yields the same final result.

There is also an established non-equivalence between a producer versus consumer levied *ad valorem* tax: in order to have the same incidence, the ad valorem tax rate, t_c, imposed on consumers must be related to the ad valorem tax rate, t_p, imposed on producers in the way that $t_p = t_c /(1+ t_c)$. If the tax rate equivalents are used, then policy objectives regarding welfare loss minimization or revenue maximization can be obtained by either administrative application of an ad valorem tax. In the literature, this equivalence result of an ad valorem tax is often pointed out by using linear demand and supply curves (see, for example, Musgrave (1959) and Rosen (2002)) and on the basis of yielding the same deadweight loss.

In this paper, we review this non-equivalence of an ad valorem tax. In so doing, we develop a formal framework in which a general proof for the non-equivalence of an ad valorem tax in terms of both deadweight loss and revenue maximization is provided. We also present some evidence showing that policy makers may be neglecting the subtlety of this equivalence in establishing revenue maximizing tax rates.

The remainder of the paper is organized as follows. In Section 2, we present our model and the results. An application that demonstrates the policy implications of the non-equivalence is provided in Section 3 and conclusions are found in Section 4.

2. The Model and Results

Consider a competitive economy with two goods: good x and the *numeraire* y. There is a representative consumer whose utility function takes the following form:

$u(x, y) = y + v(x)$

where $v' > 0$ and $v'' < 0$. Good x is produced by a representative firm with the cost function $c(x)$, where $c' > 0$ and $c'' \geq 0$[1] .

[1] We could as well start with a general decreasing demand function and a general non-decreasing supply function in a partial equilibrium setting, our results would go through without any difficulty.

Let p denote the price of good x. Then, the demand for good x is summarized by the following relation[2] :

$p = v'(x)$

and the supply of good x is described by the following[3] ,
$p = c'(x)$

The equilibrium quantity of x, to be denoted by x_0, is then determined by the following equation[4] :

$v'(x_0) = c'(x_0)$ (1)

Suppose an ad valorem tax t is imposed on good x.

(*a*) Tax is legally imposed on the consumer. In this case, the demand relation is given by

$(1+t)\, p = v'(x)$

and the supply relation is given by

$p = c'(x)$.

Therefore, the equilibrium quantity of x, to be denoted by x_1, is determined by the following equation

$(1+t)\, c'(x_1) = v'(x_1)$. (2)

(*b*) Tax is legally imposed on the producer. In this case, the demand relation is given by

$p = v'(x)$

and the supply relation is given by

$(1-t)\, p = c'(x)$.

Therefore, the equilibrium quantity of x, to be denoted by x_2, is determined by the following equation

$(1-t)\, v'(x_2) = c'(x_2)$. (3)

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[2] For the ease of presentation, we assume an interior solution.
[3] Again, we assume an interior solution.

[4] Throughout this paper, we assume the existence of an equilibrium. The same remark applies to situations where an ad valorem tax is imposed on either the consumer or the producer.

2.1. Equivalence in terms of Deadweight Loss

In this subsection, we discuss the equivalence result in terms of deadweight loss. From equations (2) and (3), we obtain

$$(1-t^2)c'(x_1)v'(x_2) = c'(x_2)v'(x_1). \quad (4)$$

Let $f(x) = c'(x)/v'(x)$. Then,

$$1-t^2 = f(x_2)/f(x_1).$$

Note that $1-t^2 < 1$. Since $c' > 0$, $v' > 0$, $v'' < 0$ and $c'' \geq 0$, clearly, $f'(x) > 0$. Therefore,

$$f(x_2) < f(x_1)$$

which implies

$$x_2 < x_1.$$

Similarly, we can show that

$$x_0 > x_1.$$

Let p_1 and p_2, respectively, be the equilibrium prices corresponding to x_1 and x_2. Then,

$$(1+t)\, p_1 = v'(x_1) \text{ and } p_2 = v'(x_2).$$

Since $v'' < 0$ and $x_1 > x_2$, it follows that

$$(1+t)\, p_1 < p_2.$$

Let L_1 and L_2, respectively, be the absolute values of deadweight loss corresponding to x_1 and x_2. Then,

$$L_1 = \int_{x1}^{x0} (v'(x) - c'(x))\, dx$$

and

$$L_2 = \int_{x2}^{x0} (v'(x) - c'(x))\, dx$$

Note that for $x < x_0$, $v'(x) - c'(x) > 0$, and $x_0 > x_1 > x_2$. It then follows that

$$L_2 > L_1.$$

Finally, let R_1 and R_2, respectively, be the amounts of tax revenues corresponding to x_1 and x_2. Then,

$R_1 = t\, p_1\, x_1$

and

$R_2 = t\, p_2\, x_2$

Therefore,

$R_1 \geq R_2$ iff $p_1 x_1 \geq p_2\, x_2$.

Note, however, that

$p_1 x_1 \geq p_2\, x_2$ if and only if $p_1\,(x_1 - x_2) + x_2\,(p_1 - p_2) \geq 0$.

Let

$\varepsilon_d = [p_2\,(x_2 - x_1)]/[x_2(p_2 - p_1)]$

be the point price elasticity of demand (from x_2 to x_1 at x_2). Then, noting that $p_2 > p_1$, we obtain

$p_1\,(x_1 - x_2) + x_2\,(p_1 - p_2) \geq 0$ if and only if $\varepsilon_d \leq - p_2/p_1$.

The following proposition summarizes the above discussion.

Proposition 1 (Non-equivalence of Ad Valorem Tax). For any given t ($0 < t < 1$), given our assumptions on the utility function $v(.)$ and the cost function $c(.)$, the following are true:

(i) $(1+t)\, p_1 < p_2$, $x_0 > x_1 > x_2$;

(ii) $L_1 < L_2$;

(iii) $R_1 \geq R_2$ *iff* $\varepsilon_d \leq - p_2/p_1$ where ε_d is the point price elasticity of demand from x_2 to x_1 at x_2 defined previously.

According to Proposition 1, for any given ad valorem tax rate t, the consumer pays a lower price and the deadweight loss is smaller if the tax is imposed on the consumer than if the tax is imposed on the producer. Furthermore, for any given t, the tax revenue can be higher if the tax is imposed on the consumer than if the tax is imposed on the producer. This would suggest that, for a given ad valorem tax rate t, the method of legally imposing tax on the consumer is superior to the method of legally imposing tax on the producer.

Given this non-equivalence result, it would be interesting to identify the exact relationship between the ad valorem tax rate t_c imposed on the consumer and the ad valorem tax rate t_p imposed on the producer in order for them to have the same tax incidence. From (2), by imposing an ad valorem tax rate t_c on the consumer, the equilibrium quantity of x, to be denoted by x_c, is determined by the following:

$$(1+ t_c)c'(x_c) = v'(x_c) \tag{5}$$

Similarly, from (3), by imposing an ad valorem tax rate t_p on the producer, the equilibrium quantity of x, to be denoted by x_p, is determined by the following:

$$(1- t_p)v'(x_p) = c'(x_p). \tag{6}$$

For the two methods to have the same tax incidence, it must be true that $x_c = x_p$. Therefore, from (5) and (6), we obtain

$$t_p = t_c /(1+ t_c),$$

which confirms the well established equivalent ad valorem tax rates imposed on producers and consumers.

The following proposition summarizes the above discussion.

Proposition 2. (Equivalence in terms of Deadweight Loss) Let t_c be the ad valorem tax rate imposed on the consumer (*tax exclusive rate)* and t_p be the ad valorem tax rate imposed on the producer (*tax inclusive rate*). If our assumptions on the utility function *v(.)* and the cost function *c(.)* are satisfied, then for the two methods to be equivalent in terms of tax incidence, it must be true that $t_p = t_c /(1+ t_c)$.

2.2 Equivalence in terms of Maximum Revenue

The non-equivalence result in Section 2.1 also gives rise to a straightforward equivalence of a revenue maximizing ad valorem tax rate on producers (t_p) or consumers (t_c), for which we turn to now.

Let t_c be the revenue-maximizing tax rate when tax is legally imposed on the consumer and t_p the revenue-maximizing tax rate when tax is legally imposed on the producer. Let $R_c = t_c\, p_c\, x_c$ be the maximum tax revenue with t_c and $R_p = t_p\, p_p\, x_p$ the maximum tax revenue with t_p. Then, t_c is the solution to the following problem:

(Problem I): $\max_t (tpx)$ s.t. [α: $p = c'(x)$] and [β: $(1+t)c'(x) = v'(x)$]

and t_p solves the following problem:

(Problem II): $\max_t (tpx)$ s.t. [γ: $p = v'(x)$] and [δ: $(1-t)v'(x) = c'(x)$]

where β is the equilibrium condition (2) and δ is the equilibrium condition (3).

We note that, by substituting the two constraints in Problem I, Problem I can be re-written as the following:

(Problem I'): $\max_x$ $(x(v'(x) - c'(x)))$

and, by substituting the two constraints in Problem II, Problem II can be re-written as follows:

(Problem II'): $\max_x$ $(x(v'(x) - c'(x)))$

Clearly, Problems I' and II' are identical and therefore they must have the following properties: $x_c = x_p$, and $R_c = R_p$. Then, from our analysis in Section 2.1, $t_p = t_c /(1+ t_c)$.

The following proposition summarizes the above discussion.

Proposition 3. (Equivalence in terms of Revenue Maximization) If our assumptions on the utility function *v(.)* and the cost function *c(.)* are satisfied, then (i) $R_c = R_p$; (ii) $t_p = t_c /(1+ t_c)$.

Therefore, the two methods of collecting taxes yield the same amount of maximum tax revenue and the corresponding ad valorem tax rates for the two methods are such that $t_p = t_c /(1+ t_c)$.

3. Application

To this point, the message of this paper is simple: for any *given* ad valorem tax rate, it matters how the tax is legally imposed; if the policy maker is to levy a given ad valorem tax rate, it is often better to impose the tax on the consumer than to impose the tax on the producer[5] . An ad valorem tax administered as a tax on the consumer or a tax on the producer *can* be made equivalent but the rate will differ in the two applications of the tax. If the tax is applied to a tax inclusive price (t_p), the equivalent tax exclusive rate would be higher (t_c).

While this non-equivalence is known in the theoretical construct and is proven more systematically above, do policy makers consider the non-equivalence of an ad valorem tax at rate *t* applied to producers or consumers? If the subtle difference in tax rates that is needed for equivalence is not considered in the actual application of the tax, the goals of a particular tax policy may be compromised. To gage whether this non-

[5] One may argue that the administrative costs of collection and compliance need to be considered in addition to the welfare cost imposed by the tax.

equivalence result is considered in the practical application of tax policy, we look to the literature on revenue maximizing tax rates.

The theoretical literature regarding calculation of revenue maximizing tax rates is clear in terms of defining a revenue maximizing tax rate under conditions related to consumer and producer behavior. In the simplest case, if we assume that there is only one good, that supply is infinitely elastic, and that the demand curve is linear, the revenue maximizing tax rate for a tax, t^*, is equal to $-1/2\eta$, where η is the own price elasticity of demand for the taxed good, and t^* is the tax exclusive (t_c) tax rate. Under less restrictive assumptions on supply and demand, the revenue maximizing rate is a function of demand and supply elasticities[6]. Since all taxes applied to a product -- excise taxes, VAT and sales taxes as well as import duties -- affect consumer demand, this calculation applies to a composite of such taxes, and not just an excise tax. The estimation of revenue-maximizing tax rates could also be expanded to include the costs of compliance and administration of the tax[7] .

It would be easy to answer the question of whether policy makers use the "correct" form of ad valorem tax (producers or consumers) to maximize revenue if we could first determine that revenue maximization were the goal of such a tax and secondly if we could obtain details on the administration of such a tax. While the latter are available for some countries, it is not typical to find documentation of the true goals of specific tax policies among countries. For this reason, we take a "back door" approach to understanding the likelihood that policy makers consider the non-equivalence of an ad valorem tax rate t and the corresponding equivalence relationship. We survey some empirical literature on revenue maximizing tax rates in developing countries and compare the revenue maximizing tax rate on producers that is equivalent to one levied on consumers. We show the difference between revenue maximizing rates based on taxing at the producer and consumer level and compare those with what is reported for a sample of countries. We believe that our results suggest that, at a minimum, the complex relationship of the equivalence between ad valorem taxes levied on consumers versus producers gives rise to different revenue maximizing tax rates make it difficult for policy makers to target the appropriate tax rates to achieve revenue maximization.

In many countries, a variety of excise taxes are levied on particular products such as alcohol, tobacco and gasoline. These products are often subject to import duties or fees as well as consumption based taxes (including VAT, retail sales and special consumption taxes). Some of these excise taxes are imposed as specific or unit taxes and are therefore not subject to an equivalence problem. We were able to find detailed administrative information for the taxation of alcohol for a set of African countries (see table below and Bird and Wallace, 2004). In most countries for which we could find

[6] A summary and derivation of the various assumptions regarding supply and demand is found in Haughton (1998).

[7] The analysis becomes much more complicated if the possible effects of other taxes such as the corporate income tax as well as compliance costs were considered. The impact of consumption changes in response to changes in excise, sales or VAT taxes is not usually taken into account: for an example of the impact of these considerations, see Bahl et al. (2003).

these data, the import duty is levied on cost plus insurance plus freight (CIF). The import duty is typically levied on the net price (exclusive of tax). In the same set of countries, for those that levy a value added tax, the tax for alcoholic products is typically levied on the retail price, inclusive of tax, but the special consumption tax is sometimes levied on the net tax price at the wholesale level.

Using these administrative details for the main taxes on alcohol, in Table 1, we show the results of calculating the revenue maximizing tax exclusive tax rate on market beer (as opposed to home brew or traditionally brewed beer) as $-1/2\eta$ (column 3). We calculate the equivalent tax-inclusive tax rate as $t_p = t_c/(1+t_c)$ (column 2)[8]. We compare these revenue maximizing tax rates (RMTRs) to the actual tax rate (column 4), given the practices of administering import duties, special consumption taxes, and sales or value added taxes. Since countries impose the taxes in a variety of ways, we use the information on the actual administration of the taxes on alcohol to adjust the actual tax rates so that they are tax exclusive equivalent rates. For example, in Tanzania the import duty is imposed at a tax exclusive rate of 25 percent (t_c), the VAT is imposed at a tax inclusive rate of 20 percent (t_p) and the excise tax is imposed at an estimated tax exclusive rate of 30 percent. The tax exclusive equivalent VAT rate is therefore 25 percent, and the overall actual tax exclusive tax rate on alcohol from these three taxes is 80 percent. This rate is lower than the revenue maximizing rate when we consider the tax inclusive basis (column 2), but higher than a revenue maximizing rate if administered on a tax inclusive basis (column 3).

In the four countries we surveyed, the wedge between these two equivalent RMTRs is substantial according to the information in Table 1.[9] Also, in most cases, the actual tax rate is quite different from the revenue maximizing rate. If policy makers choose to institute or move closer to a revenue maximizing tax rate without consideration of the non-equivalence between the tax inclusive and exclusive rates, they could over or undershoot their revenue target by a wide margin.

[TABLE 1 HERE]

4. Conclusion

In this paper, we have reviewed some established non-equivalence results of an *ad valorem* tax and provided a general proof for those results. We use data on alcohol taxes from four African countries to highlight the implications of the non-equivalence on tax policies that are meant to maximize revenue—a strategy that many countries are interested in pursuing to alleviate budgetary pressures. The results of our analysis show that, if policy makers do not consider the subtle the ad valorem rates in conjunction with

[8] We do not incorporate the impact of cascading of the various taxes. If we assume that the incidence of these taxes is on the consumer, the effective tax rates would be larger.

[9] The difference for Kenya is particularly large due to the relatively large documented elasticity of demand for market brew (Okello, 2001).

the administration of the taxes, they could impose tax rates that are not consistent with their goals. We demonstrated this in the case of revenue maximizing tax rates, but goals such as equity and efficiency are also compromised in a similar fashion.

Table 1: Differences in Equivalent Taxes for Market Beer

Country	RMTR consumers	Equivalent RMTR producers	Current tax exclusive rate (using equivalence)	Current Administration
Tanzania (Osoro, et. Al, 2001)	1.603	0.62	0.80	Import duties: 25 percent ad valorem on CIF, tax exclusive VAT: 20 percent ad valorem on retail price, tax inclusive Excise: specific (30 percent estimated effective rate, tax exclusive)
Kenya (Okello, 2001)	.0901	0.083	6.28	Import duties: 30 percent ad valorem on CIF, tax inclusive VAT: 16 percent ad valorem, tax inclusive Excise: 85 percent ad valorem, tax inclusive
South Africa (Bird and Wallace, 2003)	0.94	0.48	.676	Import duties: 25 percent ad valorem, tax inclusive VAT: 14 percent ad valorem, tax inclusive Excise: specific (18.4 percent estimated effective tax rate, tax exclusive)
Mauritius (Bird and Wallace, 2003)	1.603	0.62	1.55	Import duties: 80 percent ad valorem on value at importation, tax exclusive VAT: 15 percent ad valorem, tax exclusive Excise: specific (60 percent estimated effective tax rate, tax exclusive)

Notes: The current tax rate is the combined rate of all import, excise and VAT (or sales tax) on the product. Where rates are specific, we use the tax exclusive ad valorem rate estimate reported in Bird and Wallace (2003). To calculate the RMTR equivalence, we separate the specific taxes from the ad valorem taxes, since the specific taxes are not affected by the method of administration (consumer or producer). The RMTR are calculated using elasticities of demand as follows, Tanzania, Osoro et. Al. 2001, Kenya, Okello, 2001, South Africa, South African National Treasury, 2002, and for Mauritius, we assume the same RMTR as that of Tanzania.
CIF: Cost plus insurance plus freight.

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