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to Sectoral Analysis for Tax Potential: An
Application to Pakistan**

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A Computational General Equilibrium Approach to Sectoral Analysis for Tax Potential: An Application to Pakistan¹

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ABSTRACT

This study develops a dynamic general equilibrium model, applied to Pakistani data, in which optimizing agents evade taxes by operating in the underground economy. The cost to firms of evading taxes is that they find themselves subject to credit rationing from banks. Our model simulations show that in the absence of budgetary flexibility to adjust expenditures, raising tax rates too high drives firms into the underground economy, thereby reducing the tax base. Aggregate investment in the economy is lowered because of credit rationing. Taxes that are too low eliminate the underground economy, but result in unsustainable budget and trade deficits. Thus, the optimal rate of taxation, from a macroeconomic point of view, may lead to some underground activity. We note, in particular, that incorporating a VAT without any other tax reductions greatly reduces the tax compliance of the service sector.

We have applied our model to Pakistan, and have calibrated our model to an 8 year period from 2004-2011. We note that it gives a reasonable approximation of Pakistani macro data. We then use a sectoral breakdown of tax data generated by the model to estimate tax gaps on a sector by sector basis. We note that certain sectors are currently paying taxes below their potential, while others may be above their tax potential. These sectoral gap estimates may be used as indicators of where greater tax enforcement efforts should be directed.

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I. INTRODUCTION

In many developing and transition countries, economic activity in the underground economy is estimated in excess of 40 percent of GDP (Schneider and Enste 2000; Friedman et al. 2000).² This diversion into unofficial activity undermines the tax base and can significantly affect public finances and the quality of public administration (Loayza 1996; Johnson et al. 1997; Dessy and Pallage 2003). The illegal nature of underground activity also constrains private investment and growth. One important cost imposed by the inability to enforce legal contracts is the limited access to formal credit markets.

We develop a simple intertemporal general equilibrium model with heterogeneous agents, multiple production activities and credit rationing to explain the prevalence of a large underground economy and corresponding gap between potential and actual taxes collected. Our model is then applied to Pakistan. In particular, we explore the link between tax rates, access to credit, and the extent of tax evasion, and examine the consequences of the underground economy for public finances and aggregate economic performance. Entry and exit into the underground economy is derived as part of optimizing behavior that depends on taxes and interest rates. Firms operating underground are subject to credit rationing by banks which reduce loans in relation to the firm's nonpayment of taxes

Since the size of the underground economy in the paper depends upon both endogenous and exogenous variables, our framework has scope for policy changes. In particular, we address the issue of policy responses towards the emergence of tax-avoiding activity and emphasize the ambiguous effects of taxation by means of numerical simulations of a computational general

²As in Braun and Loayza (1993), the underground economy is defined as a set of economic units which do not comply with one or more government imposed taxes and regulations, but whose production is considered legal".

equilibrium (CGE) model for Pakistan. Economic reform will depend upon policies that reduce the various forms of tax evasion.

Section II provides a brief overview of our modeling of the underground economy. Section III presents our dynamic CGE model. Section IV discusses the parameterization of the model and presents an initial calibration exercise. Section V discusses extensions to further disaggregation of the tax gap analysis.

II. MACROECONOMIC BACKGROUND

The cost of operating in the underground economy is modeled in terms of the inability to borrow from the official banking system. Banks in the model are assumed not to have perfect information about the firm's true ownership of assets and its associated true tax obligation. We assume that due to collateral requirements, credit is provided only in relation to the firm's implied ownership of assets, which is determined from its actual tax payment. The idea here is that in the face of default, banks can only seize those assets that have been officially declared by the firm. Hence, the higher the extent of tax evasion, the lower the implied value of firm assets, and the lower the amount of credit provided by the banking system. Our approach has some similarity to Kiyotaki and Moore (1997) who model credit limits on loans. These limits are determined by estimates of collateral which, in turn, are determined by estimates of durable asset holdings by borrowers. Here, tax payments are used to estimate the value of the durable asset of the borrower, as the asset cannot be directly observed.

We assume that firms can operate partially in the formal and partially in the underground economy. That part of their operation that takes place in the legal economy pays taxes and can borrow from the banking system. That part that is underground does not pay taxes and cannot borrow. Admittedly this distinction is artificial, but captures some of the benefits and costs of

operating in the underground economy discussed in the literature. In reality, the underground firm may still be able to finance its investment needs by relying on trade credits or borrowing from secondary lenders who charge higher than market interest rates and are willing to incur high risks.³

Our approach also assumes that firms can evade taxes without any real risk of detection or punishment. Shleifer and Vishny (1993) point out that where public pressure on corruption or the enforcement ability of the government is relatively weak - as is the case in many developing countries - this is in fact a fitting assumption.

III. A DYNAMIC GENERAL EQUILIBRIUM MODEL

We have developed the formal structure of a dynamic general equilibrium model that endogenously generates an underground economy. Much of the structure of our model is designed to permit numerical implementation for Pakistan. Our model has n discrete time periods. All agents optimize in each period over a 2 period time horizon. That is, in period t they optimize given prices for periods t and $t + 1$ and expectations for prices for the future after $t + 1$. When period $t + 2$ arrives, agents re-optimize for period $t + 2$ and $t + 3$, based on new information about period $t + 2$.

Our approach is related to Gordon and Li (2009). Here the government is able to tax a firm only if that firm uses the banking system. When the firm uses a bank, it is assumed that the bank has access to the firm's balance sheet, which it records. The bank then makes this balance sheet information available to the government, which is then able to collect taxes, in particular sales taxes, based upon its knowledge of the firm's balance sheet.

³ Huq and Sultan (1991) note that in Bangladesh, while borrowing rates from commercial banks were around 12 percent, firms dependent on noninstitutional sources to meet their financing needs paid rates between 48 to 100 percent.

We use a dynamic approach in which both firms and banks optimize and in which the benefits to a firm of accessing the banking system are endogenous. Our approach is related to Dabla-Norris and Feltenstein (2005). Here a firm compares the return to capital with the marginal tax rate on capital income. If the return is greater than the tax rate, then the firm pays the full capital tax. If it is less than the tax rate, then the firm reduces its tax payments proportionally. Hence the firm enters the underground economy gradually, as the gap between tax rates and returns to capital increases. At the same time banks use a firm's capital tax payments, combined with the capital tax rate to obtain an estimate of the firm's minimum capital value. This is thus the bank's estimate of the firm's collateral, and hence reflects a minimum estimate of the value of assets that the bank can seize if the loan fails. This approach is motivated by the collateral constraints in Kiyotaki and Moore (1997). The collateral is represented by the bank's estimate of the borrowing firm's minimum capital which is, in turn, estimated by examining the firm's tax return. We should note that we are thus focusing on only a single type of tax evasion, namely, evasion of the capital income tax. As we shall see, indirect tax rates can change rates of evasion of the corporate income tax by changing the rate of return to capital. We do not, however, consider direct evasion of sales or value added taxes, for example.

Our approach has the key feature that tax evasion is based upon optimizing behavior by firms, rather than upon some exogenous firm characteristics. In particular, enterprises, as well as individuals, will balance their need to invest by borrowing from the banking system with their desire to reduce their tax obligations. This optimizing behavior is, of course, forward looking. The detailed technical discussion for production, banking, consumption, government and foreign sector is provided in Appendix 1.

IV. SIMULATIONS

In this section we carry out numerical simulations, based upon the model whose technical structure is given in Appendix 1. The model is designed to give some qualitative notion of the implications for the economy of tax evasion and entry into the underground economy. Our goal is to calibrate the model to the dynamic path of the Pakistan macro economy, based upon the most recent available sources for the economy's technological and policy parameters.

We use an input-output (IO) matrix given in Ahmad, Barrett and Coady (1985)⁴, in which an 87 sector matrix is derived to represent Pakistan's technology for 1981. This has been updated for 1989/90, and we use the coefficients in this updated matrix. This matrix is aggregated by adding rows and columns to the 27 sector matrix used for this study. Sectoral value added are taken from the national income accounts for 2004 expanded to correspond to the 27 sector IO matrix. We use 2004 as a starting point as our 8 year dynamic simulation is from 2004 – 2011. The production coefficients in sectoral value added functions are Cobb-Douglas and are taken from the IO matrix.

The model incorporates personal and corporate income taxes, sales taxes, and import tariffs. Our source for all tax rates is <http://www.taxrates.cc/html/pakistan-tax-rates.html>. For the personal income tax we use the various slabs from 0 to 20 percent. For the corporate tax rate we use 35% of net taxable income of a company. For nonresidents, a 15% rate is levied on the gross amount of royalties or technical service fees, and 30% for other payments under the presumptive tax regime. The standard rate of the sales tax in Pakistan is 16 percent. Note that these are

⁴ "Input-Output Matrices for Pakistan 1980-81." Discussion Paper, Development Economics Research Centre, University of Warwick.

statutory rather than effective rates. The model generates endogenous effective tax rates, which are different from rates generated by single equation estimates.

Table 1. Base Case

Period	2004	2005	2006	2007	2008	2009	2010	2011
Nominal GDP 1/	100.0	137.8	133.4	181.2	314.1	475.3	538.3	785.6
Real GDP 1/	100.0	117.4	113.5	119.7	131.3	142.3	144.3	149.6
Real GDP growth rate 3/		17.4	-3.3	5.4	9.7	8.4	1.4	3.6
Inflation 3/		17.4	0.1	28.9	58.0	39.7	11.6	40.8
Price Level 1/	100.0	117.4	117.5	151.4	239.2	334.1	373.0	525.3
Nominal interest rate 3/	6.9	10.5	3.6	3.6	8.4	13.1	7.7	9.7
Budget surplus 2/	-1.4	-1.3	-1.6	-1.8	1.3	-1.2	-0.9	-2.2
Trade Balance 2/	-3.7	-2.6	-2.0	0.0	-4.3	-2.0	-2.6	-1.1
Import Duties 2/	2.0	2.1	2.2	2.2	2.1	2.1	2.2	2.2

Real return to	Share of Sector in Legal Economy				
		2005	2007	2009	2011
K1 1/ 4/	100.0	100.0	100.0	100.0	100.0
K2	100.0	100.0	100.0	100.0	100.0
K3	100.0	100.0	100.0	100.0	100.0
K4	100.0	3.7	5.9	27.0	58.5
K5	100.0	100.0	100.0	100.0	100.0

1/ Normalized to period 1 of the base case.

2/ As a percent of GDP.

3/ In percent.

4/ The capital types are specific to broad sectors of the input-output matrix. The 5 capital types are:

K1 = Mining

K2 = Manufacturing

K3 = Electricity, gas, construction

K4 = Services, retail trade

K5 = Public administration, health, education

Exchange Rates time series are taken from the Statistics and DWH Department, the State Bank of Pakistan. We use the annual average US dollar foreign exchange rates for the years 2003-2010, as we wish to generate a dynamic macroeconomic path for these years. We assume that the structure of financing of the government budget deficit is an exogenous policy instrument, and we take the 2003-2010 shares from the data source TABLE 4.2, SUMMARY OF

PUBLIC FINANCE (CONSOLIDATED FEDERAL AND PROVINCIAL). We make a similar exogeneity assumption for public and private capital inflows, which are taken from the TABLE 8.1 of the same source. Our source for the historical series of expenditure by the consolidated public sector is http://www.sbp.org.pk/departments/stats/PakEconomy_HandBook/Chap-3.7.pdf where we use the shares of GDP table.

Table 4. Pakistan: 10% VAT, 0% GST

Period	2004	2005	2006	2007	2008	2009	2010	2011
Nominal GDP 1/ Real GDP	139.2	194.5	224.8	323.3	501.5	776.0	935.5	1415.2
1/	101.6	118.9	115.8	120.3	129.9	138.7	140.6	144.1
Real GDP growth rate 3/		17.0	-2.6	3.9	8.0	6.8	1.4	2.5
Inflation 3/		19.5	18.6	38.5	43.7	44.9	18.9	47.6
Price Level 1/ Nominal interest	137.0	163.6	194.1	268.7	386.1	559.5	665.4	981.8
rate3/	14.1	21.6	11.2	13.5	20.5	29.4	19.0	25.0
Budget surplus 2/	-6.5	-8.2	-8.9	-10.1	-8.0	-11.3	-10.4	-12.5
Trade Balance 2/	-5.7	-5.4	-6.7	-6.4	-8.0	-6.8	-7.7	-7.4
Import Duties 2/	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Real return to		Share of Sector in Legal Economy			
		2005	2007	2009	2011
K1 4/	100.5	100.0	100.0	100.0	100.0
K2	89.9	100.0	100.0	100.0	100.0
K3	100.4	100.0	100.0	100.0	100.0
K4	102.1	4.2	11.1	34.6	80.8
K5	105.2	100.0	100.0	100.0	100.0

1/ Normalized to period 1 of the base case.

2/ As a percent of GDP.

3/ In percent.

4/ The capital types are specific to broad sectors of the input-output matrix. The 5 capital types are:

K1 = Mining

K2 = Manufacturing

K3 = Electricity, gas, construction

K4 = Services, retail trade

K5 = Public administration, health, education

Our model incorporates behavioral demand for money that depends upon interest and inflation rates, as well as real income. We use the estimates given in Qayyum (2005). In order to use our model for counterfactual simulations, we first generate an equilibrium using benchmark policy, technological, and behavioral parameters described.⁵ The program used to solve for the equilibrium converges to an accurate approximation of a Kakutani fixed point in usually less than 20 seconds for the 8 discrete time periods we are currently simulating. We run the macroeconomic model forward for eight years,⁶ giving tax rates and public expenditures their estimated values. We also suppose that the central bank maintains a fixed exchange rate, with the rate being fixed at the historical level of each year. Table 1 shows the results of the benchmark simulation. It may be worth making a few remarks concerning the simulated values. First, notice that our model generates moderate rates of growth in real GDP, with an average growth rate of 5.9 percent over the total 8 year period. This approximates Pakistan's actual real growth rate over the period in question. The budget is in deficit for all but 1 year, with an average deficit of 1.1 percent of GDP. This is lower than the actual historical deficit for the period. The simulated interest rate is relatively stable, and averages 7.9 percent, which is in line with Pakistan's interest rate. The trade deficit is relatively stable and averages 2.3 percent of GDP, which is somewhat better than the current level in Pakistan. The annual rate of inflation averages 22.3 percent, which is somewhat higher than the Pakistani average. Finally, sector 4, services and retail trade, operates significantly in the underground economy for all 8 years of the simulation, indicating considerable tax evasion in retail trade. This also possibly corresponds to the Pakistan

⁵ The underlying computational general equilibrium program is written in FORTRAN 95 and is available by writing to Andrew Feltenstein (afeltenstein@gsu.edu).

⁶ In practice, we take 2004 as the base year. By this we mean that initial allocations of factors and financial assets are given by stocks at the end of 2003. We have data for fiscal and other policy parameters for the next 8 years, that is, through 2011.

experience. By the end of the 8 years of the simulation, the sector is under-reporting income for tax purposes by 31.5 percent.

Suppose we now consider a change in the Pakistan tax code. Namely we will impose a 10 percent value added tax, leaving all other tax rate as in Table 1. Note that here, and in the following Tables, all numbers are relative to the benchmark case of Table 1. Hence, for example, GDP and price levels change from 100 in period 1. Pakistan has considered introducing a VAT, so this will be an example of the potential outcomes of such a reform. The results are given in Table 2. We note that the tax increase leads to a small decline in real GDP over the 8 years of the simulation. The budget deficit becomes a surplus and there is also a trade surplus. At the same time, there is a considerable increase in tax evasion by sector 4 which now is evading almost 75 percent of its tax liabilities. Thus simply imposing a VAT without making other changes in tax policies would not seem to be a useful course of action.

Accordingly, let us now suppose that we maintain the 10 percent VAT, but reduce the GST from 16 to 10 percent. The aim of this exercise would be to reduce tax evasion. The results are given in Table 3. We see that real income has remained unchanged, as compared to Table 2. There has been a small improvement in tax compliance, which still remains much worse than in the base case. This outcome leads us to conclude that the VAT introduction needs to be compensated for by a more dramatic tax reduction elsewhere.

Table 2. Pakistan: 10% VAT

Period	2004	2005	2006	2007	2008	2009	2010	2011
Nominal GDP 1/ Real	99.1	136.7	121.0	164.5	262.9	398.8	427.5	611.5
GDP1/	98.2	116.5	112.1	118.6	129.8	142.4	144.3	149.1
Real GDP growth rate 3/		18.7	-3.8	5.8	9.4	9.7	1.3	3.3
Inflation 3/		16.3	-8.0	28.5	46.0	38.3	5.8	38.4
Price Level 1/ Nominal interest rate	100.9	117.3	108.0	138.7	202.5	280.0	296.3	410.2
3/	6.8	10.2	2.9	3.1	5.7	10.2	4.5	5.0
Budget surplus 2/	3.8	4.0	3.7	4.0	7.4	4.9	6.0	4.7
Trade Balance 2/	-3.6	-2.5	-0.4	1.9	-2.0	0.8	1.3	3.8
Import Duties 2/	2.0	2.1	2.2	2.2	2.1	2.0	2.1	2.1

Real return to		Share of Sector in Legal Economy			
		2005	2007	2009	2011
K1 4/	100.6	100.0	100.0	100.0	100.0
K2	108.5	100.0	100.0	100.0	100.0
K3	100.8	100.0	100.0	100.0	100.0
K4	101.1	2.1	3.0	12.2	25.2
K5	101.4	100.0	100.0	100.0	100.0

1/ Normalized to period 1 of the base case.

2/ As a percent of GDP.

3/ In percent.

4/ The capital types are specific to broad sectors of the input-output matrix. The 5 capital types are:

K1 = Mining

K2 = Manufacturing

K3 = Electricity, gas, construction

K4 = Services, retail trade

K5 = Public administration, health, education

As a fourth example, suppose we now reduce the GST to 0 percent, so that the only domestic indirect tax collected is the VAT. The results of this exercise are given in Table 4. We see that Sector 4 has greatly increased its tax compliance, even as compared to the base case of Table 1. At the same time, however, the revenues lost from the other sectors of the economy, which were not previously in the underground economy and hence did not increase their tax compliance, has outweighed the gains from Sector 4. The budget deficit thus grows dramatically, leading to a

sharp increase in the domestic interest rate. Real GDP falls, primarily because the rate of return to capital of Sector 2, manufacturing falls, leading to a decline in that sector's output.

Table 3. Pakistan: 10% VAT, 10% GST

Period	2004	2005	2006	2007	2008	2009	2010	2011
Nominal GDP 1/	105.0	145.0	140.7	193.7	316.2	479.2	539.6	791.2
Real GDP1/	98.9	117.0	113.2	119.0	130.0	140.9	143.8	149.1
Real GDP growth rate 3/		18.3	-3.2	5.1	9.2	8.4	2.0	3.7
Inflation 3/		16.7	0.3	30.9	49.5	39.7	10.4	41.4
Price Level 1/	106.2	124.0	124.3	162.7	243.3	340.0	375.3	530.6
Nominal interest rate3/	7.5	12.1	4.2	4.5	8.7	14.3	8.2	10.9
Budget surplus 2/	-0.2	-0.3	-0.3	-0.4	2.4	-0.3	0.7	-0.7
Trade Balance 2/	-3.6	-2.6	-1.9	-0.1	-3.8	-1.3	-1.7	-0.2
Import Duties 2/	1.3	1.3	1.4	1.4	1.4	1.3	1.4	1.4

Real return to		Share of Sector in Legal Economy			
		2005	2007	2009	2011
K1 4/	100.8	100.0	100.0	100.0	100.0
K2	103.4	100.0	100.0	100.0	100.0
K3	101.0	100.0	100.0	100.0	100.0
K4	101.7	2.5	4.2	17.1	36.7
K5	102.9	100.0	100.0	100.0	100.0

1/ Normalized to period 1 of the base case.

2/ As a percent of GDP.

3/ In percent.

4/ The capital types are specific to broad sectors of the input-output matrix. The 5 capital types are:

K1 = Mining

K2 = Manufacturing

K3 = Electricity, gas, construction

K4 = Services, retail trade

K5 = Public administration, health, education

Let us now consider yet another proposed policy change, namely, an increased capital tax upon the banking industry. Accordingly, we will increase the tax rate from its current 35 percent to 40 percent. All other policy parameters stay the same as in Table 1. The results of this exercise are given in Table 5. The outcomes are interesting, as we notice that there has been

very little improvement in the overall budget deficit. At the same time, we see that there has been a considerable decrease in tax compliance by the Service Sector (Sector 4), of which the banking industry is a major component. Thus the increased tax rate has caused an increase in evasion that has negated the impact of the higher tax rates on the banking sector.

Table 5. Pakistan: 40% CAPITAL TAX ON BANKS

Period	2004	2005	2006	2007	2008	2009	2010	2011
Nominal GDP 1/	100.0	137.9	132.6	178.8	312.9	472.7	538.7	782.0
Real GDP1/	100.0	117.2	113.4	118.9	131.1	141.9	143.9	148.7
Real GDP growth rate3/		17.2	-3.2	4.8	10.2	8.3	1.4	3.4
Inflation 3/		17.6	-0.6	28.7	58.7	39.5	12.4	40.5
Price Level 1/	100.0	117.6	116.8	150.4	238.6	333.0	374.4	525.8
Nominal interest rate 3/	6.8	9.8	3.1	3.1	7.2	12.1	6.6	8.2
Budget surplus 2/	-1.5	-1.0	-1.7	-1.6	1.4	-0.8	-1.2	-2.0
Trade Balance 2/	-3.5	-2.3	-1.5	0.7	-4.0	-1.5	-2.1	-0.4
Import Duties 2/	2.0	2.1	2.2	2.2	2.1	2.1	2.2	2.2

Real return to		Share of Sector in Legal Economy			
		2005	2007	2009	2011
K1 4/	98.0	100.0	100.0	100.0	100.0
K2	100.7	100.0	100.0	100.0	100.0
K3	99.9	100.0	100.0	100.0	100.0
K4	99.5	2.7	4.3	20.3	44.6
K5	99.8	100.0	100.0	100.0	100.0

1/ Normalized to period 1 of the base case.

2/ As a percent of GDP.

3/ In percent.

4/ The capital types are specific to broad sectors of the input-output matrix. The 5 capital types are:

K1 = Mining

K2 = Manufacturing

K3 = Electricity, gas, construction

K4 = Services, retail trade

K5 = Public administration, health, education

As a final exercise, let us carry out a reduction in the customs tariff, reducing the rate from 25 percent to 20 percent. This is a policy that is currently under consideration. The results of

this simulation are given in Table 6. As might be expected, we see a decline in import duties, as compared to Table 1. Tax evasion remains essentially unchanged, and there is a slight decline in real GDP, as compared to Table 1, due to the expected increase in imports. We thus see that this tariff change has little effect on the overall real economy.

Table 6. Pakistan: REDUCED CUSTOMS TARIFF

Period	2004	2005	2006	2007	2008	2009	2010	2011
Nominal GDP 1/	98.9	136.2	131.4	177.0	312.2	471.7	539.9	786.2
Real GDP 1/	99.9	117.0	113.1	118.4	131.1	142.0	144.7	149.2
Real GDP growth rate ^{3/}		17.2	-3.4	4.7	10.7	8.3	1.9	3.1
Inflation 3/		17.5	-0.2	28.6	59.3	39.5	12.3	41.3
Price Level 1/	99.0	116.4	116.2	149.5	238.2	332.2	373.1	527.1
Nominal interest rate 3/	6.4	9.7	3.0	3.0	7.2	12.0	7.2	8.9
Budget surplus 2/	-2.0	-1.5	-2.2	-2.1	1.0	-1.3	-1.6	-2.4
Trade Balance 2/	-3.4	-2.1	-1.3	1.1	-4.0	-1.4	-2.1	-0.4
Import Duties 2/	1.5	1.6	1.7	1.7	1.7	1.6	1.7	1.7

Real return to		Share of Sector in Legal Economy			
		2005	2007	2009	2011
K1 4/	98.0	100.0	100.0	100.0	100.0
K2	100.8	100.0	100.0	100.0	100.0
K3	100.0	100.0	100.0	100.0	100.0
K4	99.9	3.3	5.5	25.0	58.3
K5	99.8	100.0	100.0	100.0	100.0

1/ Normalized to period 1 of the base case.

2/ As a percent of GDP.

3/ In percent.

4/ The capital types are specific to broad sectors of the input-output matrix. The 5 capital types are:

K1 = Mining

K2 = Manufacturing

K3 = Electricity, gas, construction

K4 = Services, retail trade

K5 = Public administration, health, education

Our model helps us to identify those sectors that are underperforming from a tax point of view. We therefore use the model to carry out a sectoral estimate of the tax gap. Here the predicted outcomes of the general equilibrium model for 2010, assuming full compliance, are

compared with actual tax revenues collected. That is, we set the “honesty” parameter for each sector at 0. This is the parameter α in equation (3) in Appendix 1. The general equilibrium model then generates a path for tax collections for the 8 years of the simulation, and we choose the predicted collections for 2010. These are then compared with actual tax collections for 2010 for selected sectors, as well as for the aggregate economy and the manufacturing sector. The aggregate results are given in the Table 7 below. They indicate that, on the level of the overall economy, there is a tax gap of about 58 percent, while in the manufacturing sector the gap is approximately 53 percent. As might be expected from the general equilibrium model, capital intensive sectors such as iron and steel, and oil and gas, have smaller gaps than do less capital intensive sectors such as finance and insurance, or hotels and restaurants. These calculations should help in the measurement of the overall problem, as well as to identify those sectors where improvement is most needed.

It may be useful to add a few remarks about the absolute values of these gap estimates. Recall that the sectoral definitions of the general equilibrium model are based upon the 87 sector Pakistan input-output matrix. These sectors are, in turn, based upon national income accounts value added definitions. These sectoral definitions are not exactly the same as those in the actual tax collection tables which we use for the gap estimates. Thus, for example, the national income account definition of Finance and Insurance may be broader than that used by the tax authorities.

Accordingly, the general equilibrium model would generate greater expected tax revenue for Finance and Insurance, assuming perfect compliance, than would be reflected in actual tax collection data. Hence the estimated compliance gap would be relatively large, as we see in the Table. Of course the opposite could also be true, that the national income account definition could be narrower than the tax definition, leading to some under estimations of particular gaps.

Table 7: Tax Gaps by Selected Sectors in percent 1/

Sector	Gap
Mining & Quarrying	-96.9
Manufacturing	-52.5
(of which)	
1. Chemicals	-67.5
2. Automobiles	-48.3
3. Cigarette & Tobacco	103.4
4. Iron and Steel	-10.5
5. Oil and gas	-25.7
6. Paper & Paper Board	-53.2
7. Textile	-59.2
8. Edible Oil	75.2
9. Cement	-49.0
10. Sugar	-91.2
11. Pharmaceuticals	-46.9
12. Fertilizer	-23.0
Telecom	-81.3
Wholesale and Retail Trade	-73.4
Finance and Insurance	-93.3
Hotels and Restaurants	-85.3
Other	-53.8
Total Economy	-58.3

1/ A number of manufacturing sectors have been excluded from the disaggregation. They are included in the category "other". Other sectors that are not included in the terms of reference, but for which it is possible to calculate gaps, have been included.

Accordingly, it is best to look at broad sectors, such as the overall economy, manufacturing, or retail sales, for example, for absolute values of gaps as there is a closer comparison between national income account and tax collection definitions for these categories. For more narrowly defined sectors, it is best to look at the gap estimates as reflecting relative (compared to other sectors) rather than absolute gaps.

CONCLUSION

We have constructed a dynamic general equilibrium model that incorporates endogenous tax evasion as part of intertemporal optimizing behavior by firms. We have used parameters from Pakistan to calibrate our model to an 8 year period from 2004-2011, and note that it gives a

reasonable approximation of Pakistani macro data. We note that the service sector, at equilibrium, consistently evades some portion of its taxes.

We then incorporate a value added tax of 10 percent in a series of counterfactual simulations, as the introduction of such a tax is currently under consideration by Pakistan. We note that incorporating a VAT without any other tax reductions greatly reduces the tax compliance of the service sector. If, on the other hand, the VAT introduction is accompanied by abolishing the GST, then tax compliance rises, but revenues from the non-service sectors decline, interest rates rise, and real GDP growth slows.

We also use the model to carry out a sectoral estimate of the tax gap. Here the predicted outcomes of the general equilibrium model for 2010, assuming full compliance, are compared with actual tax revenues collected. That is, we set the “honesty” parameter for each sector at 0. The general equilibrium model then generates a path for tax collections for the 8 years of the simulation, and we choose the predicted collections for 2010. These are then compared with actual tax collections for selected sectors, as well as for the aggregate economy and the manufacturing sector. They indicate that, on the level of the overall economy, there is a tax gap of about 58 percent, while in the manufacturing sector the gap is approximately 53 percent. As might be expected from the general equilibrium model, capital intensive sectors such as iron and steel, and oil and gas, have smaller gaps than do less capital intensive sectors such as finance and insurance, or hotels and restaurants. These calculations should help in the measurement of the overall problem, as well as to identify those sectors where improvement is most needed. Our approach demonstrates a method for gap analysis which can be further developed to incorporate additional information on sectoral structures and use sectoral estimates of honesty parameters.

APPENDIX 1

A General Equilibrium Specification

In this section we develop the formal structure of a dynamic general equilibrium model that endogenously generates an underground economy. Much of the structure of our model is designed to permit numerical implementation for Pakistan. Our model has n discrete time periods. All agents optimize in each period over a 2 period time horizon. That is, in period t they optimize given prices for periods t and $t + 1$ and expectations for prices for the future after $t + 1$. When period $t + 2$ arrives, agents reoptimize for period $t + 2$ and $t + 3$, based on new information about period $t + 2$.

Production

There are 8 factors of production and 3 types of financial assets:

- | | | | |
|-----|---------------|-----|-------------------|
| 1-5 | Capital types | 9. | Domestic currency |
| 6. | Urban labor | 10. | Bank deposits |
| 7. | Rural labor | 11. | Foreign currency |
| 8. | Land | | |

The five types of capital correspond to five aggregate nonagricultural productive sectors.⁷

An input-output matrix, A_t , is used to determine intermediate and final production in period t .

The matrix is 27×27 , using the disaggregation of Ahmad, Barrett and Coady (1985).

Corresponding to each sector in the input-output matrix, sector-specific value added is produced using capital and urban labor for the nonagricultural sectors, and land and rural labor in agriculture.

The specific formulation of the firm's problem is as follows. Let y_{Ki}^j, y_{Li}^j be the inputs of capital and urban labor to the j th nonagricultural sector in period i . Let Y_{Gi} be the outstanding

⁷ We could have any number of capital types without affecting the structure of the model.

stock of government infrastructure in period i . The production of value added in sector j in period i is then given by:

$$va_{ji} = va_{ji}(y_{Ki}^j, y_{Li}^j, Y_{Gi}) \quad (1)$$

where we suppose that public infrastructure may act as a productivity increment to private production.

Sector j pays income taxes on inputs of capital and labor, given by t_{Kij} , t_{Lij} respectively, in period i . The interpretation of these taxes is that the capital tax is a tax on firm profits, while the labor tax is a personal income tax that is withheld at source.

We suppose that each type of sectoral capital is produced via a sector-specific investment technology that uses inputs of capital and labor to produce new capital. Investment is carried out by the private sector and is entirely financed by domestic borrowing

Let us define the following notation:

C_{Hi} = The cost of producing the quantity H of capital of a particular type in period i .

r_i = The interest rate in period i .

P_{Ki} = The return to capital in period i .

P_{Mi} = The price of money in period i .

δ_i = The rate of depreciation of capital.

Suppose, then, that the rental price of capital in period 1 is P_1 . If C_{H1} is the cost-minimizing cost of producing the quantity of capital, H_1 , then the cost of borrowing must equal the present value of the return on new capital. Hence:

$$C_{H1} = \sum_{i=2}^n \left[\frac{P_{Ki}(1-\delta)^{i-2} H_1}{\prod_{j=1}^{i-1} (1+r_j)} \right] \quad (2)$$

where r_j is the interest rate in period j , given by:

$$r_j = 1/P_{Bj}$$

where r_j is the price of a bond in period j . The tax on capital is implicitly included in the investment problem, as capital taxes are paid on capital as an input to production.

The to invest depends not only on the variables in the above equation, but also upon the decision the firm makes as to whether it should pay taxes.⁸ This decision determines the firm's entry into the underground economy. We assume that the firm's decision is based upon a comparison of the tax rate on capital with the rate of return on new capital. Formally, suppose that we were in a two period world. Suppose that:

$$\frac{P_{K2}}{1+r_1} \geq t_{K1}$$

that is, the present value of the return on one unit of new capital is greater than the current tax rate on capital. In this case we assume the investor pays the full tax rate on capital inputs.

Suppose, on the other hand, that:

$$\frac{P_{K2}}{1+r_1} \leq t_{K1}$$

Here the discounted rate of return is less than the tax rate. The extent to which the firm goes into the underground economy is determined by the gap between the tax rate and the rate of return to investment. That is, the firm pays a tax rate of \bar{t}_{K1} where:

⁸ In reality, we can regard the tax rate on capital as the generalized tax rate, including taxation, regulation, and corruption (bribes).

$$\bar{t}_{K1} = t_{K1} \left[1 - \frac{\left(t_{K1} - \frac{P_{K2}}{1+r_2} \right)}{t_{K1}} \right]^\alpha \quad (3)$$

Here $0 \leq \alpha$ and higher values of α lead to lower values of taxes actually paid. That is, the ratio $\frac{\bar{t}_{K1}}{t_{K1}}$ reflects the share of the sector that operates in the above ground economy. Hence α represents a firm-specific behavioral variable. An “honest” firm would set $\alpha = 0$, while a firm that is prone to evasion would have a high value for α .

Banking

We will suppose that there is one bank for each nonagricultural sector of the economy. There are 5 such sectors, and hence 5 banks, corresponding to each of the aggregate capital stocks. Each bank lends primarily to the sector with which it is associated. The banks are, however, not fully specialized in the sector they correspond to. We make the simplifying assumption that each bank holds a fixed share of the outstanding debt of its particular sector. It then holds additional fixed shares of the debt of each of the remaining sectors. We make this assumption of diversification of assets in order to allow for a situation in which a firm that evades taxes, and thereby enters the underground economy, might receive varying degrees of credit rationing from the different banks to which it applies for loans.

Our premise is that banks have no direct way of knowing whether specific firms operate in the underground economy. We assume that banks only care about the amount of capital that they estimate the firm may have. If the firm defaults on its loan, then this represents the best estimate of the amount that the bank could seize. The bank would, presumably, be willing to lend an amount equal to at least the estimated firm capital.

We assume the borrower is required to show the bank his tax returns in order to obtain a loan. There is a single, flat corporate tax rate that the borrowing firm faces. Hence, suppose that T_{K1} represents taxes actually paid by the borrower in period 1. This is known to the bank, as the potential borrower is required to present his tax returns. Thus if the borrower fully complied with his tax obligation, and hence carried out no underground activity, the value of his capital, \hat{K}_1 , would be given by:

$$\hat{K}_1 = \frac{T_{K1}}{t_{K1}}$$

In this case the bank lends an amount L_1 , where $L_1 < C_{H1}$, as the bank would not be able to seize the full value of the loan in the case of a default. The situation we have described would, in the case of perfect certainty, have credit rationing when the estimated value of the firm's capital is less than its loan request. If the firm's capital is greater than its loan request, there would be no credit rationing.

In a more realistic case of uncertainty about both the true value of the firm, as well as about the bank's own ability to seize the firm, one might expect the lending process to be somewhat different. Accordingly, we will suppose that a simple functional form determines bank lending as a function of the amount requested as well as the estimated value of the firm's capital. We define the amount the bank lends, L_1 , as:

$$L_1 = C_{H1} \left[\frac{\frac{\hat{K}_1}{C_{H1}}}{1 + \frac{\hat{K}_1}{C_{H1}}} \right]^\gamma = C_{H1} \left[\frac{\hat{K}_1}{C_{H1} + \hat{K}_1} \right] \quad (4)$$

Here γ represents a measure of risk aversion by the bank. If $\gamma = 0$, there are no credit restrictions, and the bank ignores estimates of the borrower's estimated net worth. As γ rises,

the bank increasingly restricts lending if the term in brackets is less than 1. Thus if a firm operates entirely in the underground economy it will not be able to borrow to finance investment. If banks are highly risk averse, they will never lend more than a firm's estimated net worth, which is based on its tax return. This tax return therefore represents all the information the bank needs in order to determine its response to a request for a loan.

Consumption

There are two types of consumers, representing rural and urban labor. We suppose that the two consumer classes have differing Cobb-Douglas demands and endowments. The consumers maximize intertemporal utility functions, which have as arguments the levels of consumption and leisure in each of the two periods.

Formally, the consumer's problem is then given by equation 5.⁹ The definition of the notation follows.

$$\text{Max } U(x), \quad x = (x_1, x_{Lu1}, x_{Lr1}, x_2, x_{Lu2}, x_{Lr2}) \quad (5)$$

such that:

$$(1 + t_i)P_i x_i + P_{Lui} x_{Lui} + P_{Lri} x_{Lri} + P_{Mi} x_{Mi} + P_{Bi} x_{Bi} + e_i P_{Bfi} x_{Bfi} = C_i \quad (5a)$$

$$P_{K1} K_0 + P_{A1} A_0 + P_{Lu1} L_{u1} + P_{Lr1} L_{r1} + P_{M1} M_0 + r_0 B_0 + P_{B1} B_0 + e_1 P_{BF1} B_{F0} + TR_1 = N_1$$

$$P_{K2}(1 - \delta)K_0 + P_{A2} A_0 + P_{Lu2} L_{u2} + P_{Lr2} L_{r2} + P_{M2} x_{M1} + r_1 x_{B1} + e_2 P_{BF2} x_{BF1} + TR_2 = N_2$$

$$C_i = N_i$$

$$\log P_{Bi} x_{Bi} - \log e_i P_{Bfi} x_{Bfi} = \alpha + \beta(\log r_i - \log \frac{e_{i+1}}{e_i} r_{Fi}) \quad (5b)$$

$$\log(L_{ui} / L_{ri}) = a_1 + a_2 \log \frac{P_{Lui} - P_{Lri}}{P_{Lui} + P_{Lri}} \quad (5c)$$

⁹ See Feltenstein and Shamloo (2012) for a discussion of this modeling approach.

$$\log P_{M_i} x_{M_i} = a + b \log(1 + t_i) P_i x_i \quad (5d)$$

$$P_{B_2} x_{B_2} = d_0 + d_1(1 + t_2) P_2 x_2 + d_2 \left[\frac{r_2 - \pi_2}{1 + \pi_2} \right] \quad (5e)$$

where:

P_i = price vector of consumption goods in period i .

x_i = vector of consumption in period i .

C_i = value of aggregate consumption in period i (including purchases of financial assets).

N_i = aggregate income in period i (including potential income from the sale of real and financial assets).

t_i = vector of value added tax rates in period i .

$P_{L_{ui}}$ = price of urban labor in period i .

L_{ui} = allocation of total labor to urban labor in period i .

$x_{L_{ui}}$ = demand for urban leisure in period i .

$P_{L_{ri}}$ = price of rural labor in period i .

L_{ri} = allocation of total labor to rural labor in period i .

$x_{L_{ri}}$ = demand for rural leisure in period i .

a_2 = elasticity of rural/urban migration.

P_{K_i} = price of capital in period i .

K_0 = initial holding of capital.

P_{A_i} = price of land in period i .

A_0 = initial holding of land.

δ = rate of depreciation of capital.

P_{Mi} = price of money in period i . Money in period 1 is the numeraire and hence has a price of 1.

x_{Mi} = holdings of money in period i .

P_{Bi} = discount price of a certificate of deposit in period i .

π_i = domestic rate of inflation in period i .

r_i, r_{Fi} = the domestic and foreign interest rates in period i .

x_{Bi} = quantity of bank deposits, that is, CD's in period i .

e_i = the exchange rate in terms of units of domestic currency per unit of foreign currency in period i .

x_{BFi} = quantity of foreign currency held in period i .

TR_i = transfer payments from the government in period i .

a, b, α, β = estimated constants.

d_i = constants estimated from model simulations.

The Government

The government collects personal income, corporate profit, and value-added taxes, as well as import duties. It pays for the production of public goods, as well as for subsidies. In addition, the government must cover both domestic and foreign interest obligations on public debt. The resulting deficit is financed by a combination of monetary expansion, as well as domestic and foreign borrowing

The Foreign Sector

The foreign sector is represented by a simple export equation in which aggregate demand for exports is determined by domestic and foreign price indices, as well as world income. The specific form of the export equation is:

$$\Delta X_{n0} = \sigma_1 \left[\frac{\pi_1}{\Delta e_i + \pi_{Fi}} \right] + \sigma_2 \Delta y_{wi}$$

where the left-hand side of the equation represents the change in the dollar value of exports in period i , π_i is inflation in the domestic price index, Δe_i is the percentage change in the exchange rate, and π_{Fi} is the foreign rate of inflation. Also, Δy_{wi} represents the percentage change in world income, denominated in dollars. Finally, σ_1 and σ_2 are corresponding elasticities.

REFERENCES

Ahmad, E., S. Barrett and D. Coady (1985), *Input-output matrices for Pakistan, 1980-81*, Issue 68 of Development Economics Research Centre. Discussion Paper, University of Warwick. Development Economics Research Centre.

Ahmed, M. A. and Masood, Q. (1995) Estimation of the Black Economy of Pakistan through the Monetary Approach, *The Pakistan Development Review* 34: 791–807.

Ahmad, R. and Rider, M. (2008) Pakistan's Tax Gap: Estimates by Tax Calculation and Methodology, ISP Working Paper, 08-11, Andrew Young School of Policy Studies, Georgia State University, Atlanta.

Ball, S., and Feltenstein, A. (2001) Bank failures and fiscal austerity: policy prescriptions for a developing country, *Journal of Public Economics* 82: 247–270.

Blejer, M. *et al.* (2002) Exogenous shocks, contagion, and bank soundness: a macroeconomic framework, *Journal of International Money and Finance* 21: 33–52.

Braun, J. and Loayza, N. (1993) Taxation, public services, and the informal sector in a model of endogenous growth (unpublished, Harvard University).

Burgess, R. and Stern, N. (1993) Taxation and development, *Journal of Economic Literature* 31: 762–830.

Dabla-Norris, Era, and Andrew Feltenstein (2005), “The Underground Economy and its Macroeconomic Consequences”, *Journal of Policy Reform* 8(2): 153-174.

De Soto, H. (1989) *The Other Path* (New York: Harper and Row).

Dessy, S. and Pallage, S. (2003) Taxes, inequality and the size of the informal sector, *Journal of Development Economics*, 70(1): 225–233.

Feltenstein, A. and Shah, A. (1993) General equilibrium effects of taxation on investment in a developing country: the case of Pakistan, *Public Finance*, 48(3): 366–386.

Feltenstein, A. and M. Shamloo (2012), “Tax Reform, the Informal Economy, and Bank Financing of Capital Formation,” forthcoming, *International Tax and Public Finance*.

Friedman, E. *et al.* (2000) Dodging the grabbing hand: the determinants of unofficial activity in 69 countries, *Journal of Public Economics* 76: 459–493.

Gordon, Roger, and Wei Li (2009), “Tax Structures in Developing Countries: Many Puzzles and a Possible Explanation”, *Journal of Public Economics* 93: 855-866.

<http://www.taxrates.cc/html/pakistan-tax-rates.html>

http://www.sbp.org.pk/departments/stats/PakEconomy_HandBook/Chap-3.7.pdf

Gorodnichenko, Y., Martinez-Vazquez, J. and Sabirianova-Peter, K. (2009). Myth and Reality of Flat Tax Reform: Micro Estimates of Tax Evasion Response and Welfare Effects in Russia, *Journal of Political Economy* 117(3): 504-554

Johnson, S. *et al.* (1997) The unofficial economy in transition, Brookings Paper on Economic Activity, Brookings Institute, pp. 159–239.

Johnson, S. *et al.* (1998) Corruption, public finance and the unofficial economy, ECLAC conference.

Johnson, S. *et al.* (2000) Why do firms hide?, *Journal of Public Economics* 76: 495–520.

Kiyotaki, N. and Moore, J. (1997) Credit cycles, *Journal of Political Economy*, 105(2): 211–48.

Kydland, F. and Prescott, E. (1977) Rules rather than discretion: the inconsistency of optimal plans, *Journal of Political Economy*, 85(3): 473–491.

Loayza, N. (1996) The economics of the informal sector and some empirical evidence from Latin America, *Carnegie Rochester Conference on Public Policy*, 45: 129–162.

Martinez-Vazquez, J. and Richter, K. (2009). National Tax Policy Reform, Chapter 2, in Martinez-Vazquez, J. and Richter, K. (eds.), *Pakistan Tax Policy Report: Tapping Tax Based for Development*, Andrew Young School of Policy Studies, Georgia State University, Atlanta.

McMillan, J. and Woodruff, C. (1999) Interfirm relationships and informal credit in Vietnam, *Quarterly Journal of Economics*, 114: 1285–1320.

Piggott, John, and John Whalley (2001), “VAT Base Broadening, Self Supply, and the Informal Sector,” *American Economic Review*, 91(4): 1084-1094.

Qayyum, Abdul, “Modelling the Demand for Money in Pakistan”, Pakistan Institute of Development Economics, 2005, Online at <http://mpr.ub.uni-muenchen.de/2057/>, MPRA Paper No. 2057, posted 07. November 2007 / 02:12

Sarte, P. (2000) Informality and rent-seeking bureaucracies in a model of long-run growth, *Journal of Monetary Economics* 46: 173–197.

Schneider, F. and Enste, D. (2000) Shadow economies: sizes, causes and consequences, *Journal of Economic Perspectives* 38: 77–114.

Shleifer, A. and Vishny, R. (1993) Corruption, *Quarterly Journal of Economics*, 108(3): 599–618.

Shliefer, A. (1997) Schumpeter lecture: government in transition, *European Economic Review* 41(3): 385–410.

Statistics and DWH Department, the State Bank of Pakistan

Straub, S. (2003) Informal sector: the credit market channel, Department of Economics Working Paper, University of Edinburgh.

Strotz, R. (1956) Myopia and inconsistency in dynamic utility maximization, *Review of Economic Studies* 23(3): 165–180

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