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Rethinking the Political Economy of Decentralization: How Elections and Parties Shape the Provision of Local Public Goods

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Appendix – For Online Publication

Rethinking the Political Economy of Decentralization: How Elections and Parties Shape the Provision of Local Public Goods

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Lemma 1 *Local public goods with and without spillovers are Pareto efficient for an economy with a majoritarian electoral system, single member districts, a democratically centralized government, and a centralized party system. All parties converge in providing a uniform local public good across districts,* $g_c^{*zi} = g_c^{*z,-i} = g_c^* \;\forall\, z$ *satisfying*

$$\int_{\forall\, e^i} h^i(e^i)\frac{\partial \mu^i}{\partial G^i}de^i + k^i \int_{\forall\, e^{-i}} h^{-i}(e^{-i})\frac{\partial \mu^{-i}}{\partial G^{-i}}de^{-i} = \left\{\frac{1}{N}\right\}\sum_{\forall i,-i}\int_{\forall\, e^i} h^i(e^i)\frac{\partial \mu^i}{\partial x^i}de^i \qquad (A.1)$$

Proof

The parties' problem is $Max\ \delta_c^z(\mathbf{g_c^z}, \xi_c^z) = \pi_c^z(\rho_c^z)$ subject to $g_c^{zi} = g_c^{z,-i} = g_c^z$. We impose the equality restriction in the objective function of party z. The first order conditions is $\frac{\partial \delta_c^z}{\partial g_c^z} = \frac{\partial \phi_c^z}{g_c^z} = \sum_{\forall\, i,-i}\int_{\forall\, e^i} h^i(e^i)\frac{\partial F_c^{zi}(\Psi_c^{zi})}{\partial \Psi_c^{zi}}\frac{\partial v^{zi}}{\partial G^{zi}}\frac{\partial G^{zi}}{\partial g_c^z}de^i = 0 \;\; \forall g_c^{*z} > 0$.The parties' policies converge, $g_c^{*z} = g_c^{*,-z} = g_c^*$, in probabilistic voting models with homogeneous parties (see Coughlin 1992) hence $f_c^{zi}(0) = f_c^{z,-i}(0) \in \mathbb{R}_+ \;\forall\, i, \forall\, z$. Therefore the first order condition becomes $\sum_{\forall\, i,-i}\int_{\forall\, e^i} h^i(e^i)\frac{\partial v^i}{\partial G^i}\frac{\partial G^i}{\partial g_c}de^i = 0$. Use $\frac{\partial v^i}{\partial G^i}\frac{\partial G^i}{\partial g_c} = (1 + k^{-i})\frac{\partial \mu^i}{\partial G^i} - \left(\frac{1}{N}\right)\frac{\partial \mu^i}{\partial x^i}$ and $\frac{\partial v^i}{\partial G^{-i}}\frac{\partial G^{-i}}{\partial g_c} = (1 + k^i)\frac{\partial \mu^{-i}}{\partial G^{-i}} - \left(\frac{1}{N}\right)\frac{\partial \mu^{-i}}{\partial x^{-i}}$ to show that the uniform local public good g_c^* satisfies

$$\int_{\forall\, e^i} h^i(e^i)\frac{\partial \mu^i}{\partial G^i}de^i + k^i \int_{\forall\, e^{-i}} h^{-i}(e^{-i})\frac{\partial \mu^{-i}}{\partial G^{-i}}de^{-i} = \left\{\frac{1}{N}\right\}\sum_{\forall i,-i}\int_{\forall\, e^i} h^i(e^i)\frac{\partial \mu^i}{\partial x^i}de^i \qquad (A.2)$$

■

Lemma 2 *Party centralization in a system of local governments leads to a set of Pareto efficient local public goods with and without spillovers* $\mathbf{g_{cL}^*} = [g_{cL}^{*i}, g_{cL}^{*-i}]$. *At the political equilibrium,* $g_{cL}^{*i}\ \forall i, \forall z$ *satisfies the following:*

$$\int_{\forall\, e^i} h^i(e^i)\frac{\partial \mu^i}{\partial G^i}de^i + k^{-i} \int_{\forall\, e^{-i}} h^{-i}(e^{-i})\frac{\partial \mu^{-i}}{\partial G^{-i}}de^{-i} = \left\{\frac{1}{N}\right\}\int_{\forall\, e^i} h^i(e^i)\frac{\partial \mu^i}{\partial x^i}de^i \qquad (A.3)$$

Proof

In the local election of district i party z selects $g_{cL}^{*zi} \in \arg\max\ \pi_{cL}^{z} = \pi_{cL}^{z}\left(\rho_{cL}^{zi}, \rho_{cL}^{z,-i}\right)$. The first order condition for an interior maximizer with $g_{cL}^{*zi} > 0$ is $\frac{\partial \pi_{cL}^{z}}{\partial \rho_{cL}^{zi}} \frac{\partial \rho_{cL}^{zi}}{\partial g_{cL}^{zi}} + \frac{\partial \pi_{cL}^{z}}{\partial \rho_{cL}^{z,-i}} \frac{\partial \rho_{cL}^{z,-i}}{\partial g_{cL}^{zi}} = 0$. By definition $\rho_{cL}^{zi} = \phi_{cL}^{zi} - \phi_{cL}^{-zi}$ and the sum of the expected proportion of the votes for parties z and $-z$ is 1, that is, $\phi_{cL}^{zi} + \phi_{cL}^{-zi} = 1\ \forall\, i$ therefore $\frac{\partial \rho_{cL}^{zi}}{\partial g_{cL}^{zi}} = 2 \frac{\partial \phi_{cL}^{zi}}{\partial g_{cL}^{zi}}$ for g_{cL}^{*zi} $\forall i, \forall z$. Then $\frac{\partial \phi_{cL}^{zi}}{\partial g_{cL}^{zi}} = \int_{\forall\, e^{i}} h^{i}\left(e^{i}\right) f_{cL}^{zi} \left\{\frac{\partial \mu^{i}}{\partial G^{i}} - \left(\frac{1}{N}\right) \frac{\partial \mu^{i}}{\partial x^{i}}\right\} de^{i}$ and $\frac{\partial \phi_{cL}^{z,-i}}{\partial g_{cL}^{zi}} = k^{i} \int_{\forall\, e^{-i}} h^{-i}\left(e^{-i}\right) f_{cL}^{z,-i} \frac{\partial \mu^{z,-i}}{\partial G^{zi}} de^{-i}$. The convergence of the parties' policies $g_{cL}^{*zi} = g_{cL}^{*-zi} = g_{cL}^{*i}$, $\forall\, z, -z \in i$ implies $f_{cL}^{zi}(0) = f_{cL}^{z,-i}(0) \in \mathbb{R}_{+}$, $\rho_{cL}^{zi} = \rho_{cL}^{z,-i} = 0\ \forall\, i, \forall\, z$. Define $\Theta^{i} = \frac{\partial \pi_{cL}^{z}(0)}{\partial \rho_{cL}^{z,-i}} \Big/ \frac{\partial \pi_{cL}^{z}(0)}{\partial \rho_{cL}^{zi}} = 1, \forall i$ therefore $\frac{\partial \pi_{cL}^{z}}{\partial \rho_{cL}^{zi}} \frac{\partial \rho_{cL}^{zi}}{\partial g_{cL}^{zi}} + \frac{\partial \pi_{cL}^{z}}{\partial \rho_{cL}^{z,-i}} \frac{\partial \rho_{cL}^{z,-i}}{\partial g_{cL}^{zi}} = 0$ is equivalent to

$$\int_{\forall\, e^{i}} h^{i}\left(e^{i}\right) \frac{\partial \mu^{i}}{\partial G^{i}} de^{i} + k^{-i} \int_{\forall\, e^{-i}} h^{-i}\left(e^{-i}\right) \frac{\partial \mu^{-i}}{\partial G^{-i}} de^{-i} = \left\{\frac{1}{N}\right\} \int_{\forall\, e^{i}} h^{i}\left(e^{i}\right) \frac{\partial \mu^{i}}{\partial x^{i}} de^{i} \qquad (A.4)$$

■

Theorem 1 ***"Strong Decentralization Theorem".*** *The provision of local public goods with and without inter-regional spillovers by a system of local governments welfare-dominates their centralized provision when party systems are centralized.*

Proof

It is simple to verify that conditions (2), (4) and (6) imply $\hat{\mathbf{g}}^{*} = \mathbf{g}_{\mathbf{cL}}^{*} \neq \mathbf{g}_{\mathbf{c}}^{*}$ for $\hat{\mathbf{g}}^{*}, \mathbf{g}_{\mathbf{cL}}^{*}, \mathbf{g}_{\mathbf{c}}^{*} \in \mathbb{R}^{2}$: $\hat{\mathbf{g}}^{*} = \left[\hat{g}^{*i}, \hat{g}^{*,-i}\right]$, $\mathbf{g}_{\mathbf{cL}}^{*} = \left[g_{cL}^{*i}, g_{cL}^{*-i}\right]$ and $\mathbf{g}_{\mathbf{c}}^{*} = \left[g_{c}^{*i}, g_{c}^{*-i}\right]$. By the strict concavity of $NSW = \sum_{\forall\, i,-i} \int_{\forall\, e^{i}} h^{i}\left(e^{i}\right) v^{i}\left(e^{i}, G^{i}\right) de^{i}$ on the constrained policy space $\exists$ feasible $\mathbf{g^{o}} = \left[g^{i0}, g^{-i0}\right], \mathbf{g^{1}} = \left[g^{i1}, g^{-i1}\right], \Omega \in [0,1]$: $\Omega = \frac{1}{g^{i1} - g^{io}} = \frac{1}{g^{-i1} - g^{-io}}$ and $\mathbf{g}^{\mathbf{\Omega}} = \Omega \mathbf{g^{1}} + (1 - \Omega)\mathbf{g^{o}}$ such that $NSW(\mathbf{g}^{\mathbf{\Omega}}) > \{\Omega NSW(\mathbf{g^{1}}) + (1 - \Omega)\mathrm{NSW}(\mathbf{g^{o}})\}$**.** Note $\Omega NSW(\mathbf{g^{1}}) + (1 - \Omega)\mathrm{NSW}(\mathbf{g^{o}}) = \mathrm{NSW}(\mathbf{g^{o}}) + \Omega\{NSW(\mathbf{g^{1}}) - \mathrm{NSW}(\mathbf{g^{o}})\}$ which implies $NSW(\mathbf{g}^{\mathbf{\Omega}}) - NSW(\mathbf{g^{o}}) >$

$\Omega\{NSW(\mathbf{g^1}) - \text{NSW}(\mathbf{g^o})\}$. We take $Lim_{\{g^{i1}-g^{io}\}_{\forall i}\to 0}\ \Omega\{NSW(\mathbf{g^1}) - \text{NSW}(\mathbf{g^o})\} = \frac{\partial NSW(\mathbf{g})}{\partial g^i} + \frac{\partial NSW(\mathbf{g})}{\partial g^{-i}}$ to show

$$NSW(\hat{\mathbf{g}}^*) - NSW(\mathbf{g}) > \frac{\partial NSW(\mathbf{g})}{\partial g^i} + \frac{\partial NSW(\mathbf{g})}{\partial g^{-i}} \qquad (A.5)$$

Without of loss of generality, set $\mathbf{g^\Omega} = \hat{\mathbf{g}}^* = \left[\hat{g}^{*i}, \hat{g}^{*,-i}\right]$, where $\hat{g}^{*i}$ and $\hat{g}^{*,-i}$ are global maximizers of NSW and $\mathbf{g^o}$ is some feasible $\mathbf{g} = \left[g^i, g^{-i}\right]: \mathbf{g} \neq \hat{\mathbf{g}}^*$ then it is satisfied that $\frac{\partial NSW(\hat{\mathbf{g}}^*)}{\partial g^i} = \frac{\partial NSW(\hat{\mathbf{g}}^*)}{\partial g^{-i}} = 0\ \forall\ \hat{g}^{*i} > 0\ \forall i \implies NSW(\hat{\mathbf{g}}^*) > NSW(\mathbf{g})\quad \forall\ g^i \neq \hat{g}^{*i}, \forall i.$ Therefore conditions (2), (4) and (6) mean $\mathbf{g^*_{cL}} = \hat{\mathbf{g}}^*$ and $\mathbf{g^*_c} \neq \hat{\mathbf{g}}^*$ and hence $NSW(\mathbf{g^*_{cL}}) > NSW(\mathbf{g^*_c})$. ■

Lemma 3 *For economies with a decentralized party system and democratic centralization a uniform and Pareto efficient local public good $g^i = g^{-i} = g_d^{*z}\ \forall\, z$ is provided such that it satisfies the following:*[1]

$$\sum\nolimits_{\forall\, i,-i} \alpha^{zi} \int_{\forall\, e^i} h^i(e^i)\, \upsilon_g^{zi}(g_d^{*z}) de^i$$
$$= -\gamma^z \left(\sum\nolimits_{\forall\, i,-i} \int_{\forall\, \tilde{e}^i} \tilde{h}^i(\tilde{e}^i) \frac{\partial \tilde{F}_0^{zi}}{\partial \tilde{\Psi}_0^{zi}} \tilde{\upsilon}_g^{zi}(g_d^{*z}) d\tilde{e}^i \right) - \sigma_\omega^z \qquad (A.6)$$

Where $\upsilon_g^{zi} = \{\partial\mu^{zi}/\partial G^i\} - \left\{\frac{1}{N}\right\}\{\partial\mu^{zi}/\partial x^i\}\ \forall zi$. *Moreover,* $\alpha^{zi} \in (0,1)$:

$$\alpha^{zi} = \sum_{l=\{1,2\}} \frac{\partial \Pi_d^z}{\partial \rho_l^z} \int_{\forall\, e^i} h^i(e^i) \frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}} de^i \Bigg/ \sum_{\forall i,-i} \sum_{l=\{1,2\}} \frac{\partial \Pi_d^z}{\partial \rho_l^z} \int_{\forall\, e^i} h^i(e^i) \frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}} de^i \qquad (A.7)$$

$$\gamma^z = \frac{\partial \Pi_d^z}{\partial \tilde{\rho}_0^z} \Bigg/ \sum_{\forall i,-i} \sum_{l=\{1,2\}} \frac{\partial \Pi_d^z}{\partial \rho_l^z} \int_{\forall\, e^i} h^i(e^i) \frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}} de^i \qquad (A.8)$$

γ^z is a weighted rate of substitution between marginal changes in the parties´ plurality in the primary and the general election, and

[1] Note that equations (A.6) to (A.9) are conditions (8) to (11) in the paper.

$$\boldsymbol{\sigma_{\omega}^{z}} = \sum_{\forall i,-i}\sum_{l=\{1,2\}} \frac{\partial \Pi_d^z}{\partial \rho_l^z} \sigma_l^{zi}\left(\frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}}, \frac{\partial \Psi_l^{zi}}{\partial g_d^z}\right) \Bigg/ \sum_{\forall i,-i}\sum_{l=\{1,2\}} \frac{\partial \Pi_d^z}{\partial \rho_l^z} \int_{\forall\, e^i} h^i(e^i)\frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}} de^i \qquad (A.9)$$

Where σ_w^z is a weighted covariance between the marginal probability of voting for party z in the nationwide general election, $\frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}}$, and the change in well being of voters from an increase in the provision of the local public good $\frac{\partial \Psi_l^{zi}}{\partial g^z}$.

Proof

For an economy with party decentralization and a single government, party z designs public spending to maximize $\Pi_d^{jz}(\tilde{\rho}_0^{jz}, \rho_l^{jz})$ subject to $g_d^{zi} = g_d^{z,-i} = g_d^z$. We impose the equality restriction in the objective function of party z. The first order condition for the party's problem is

$$\frac{\partial \Pi_d^z}{\partial \tilde{\rho}_0^z}\frac{\partial \tilde{\phi}_0^z}{\partial g_d^z} + \sum_{l=\{1,2\}} \frac{\partial \Pi_d^z}{\partial \rho_l^z}\frac{\partial \phi_l^z}{\partial g_d^z} = 0 \quad \forall\, g_d^{*z} > 0, \forall z, \forall i \qquad (A.10)$$

By definition $\tilde{\phi}_0^z = \sum_{\forall i,-i} \tilde{\phi}_0^{zi}$, $\phi_l^z = \sum_{\forall i,-i} \phi_l^{zi}$ for $l = 1,2$. The sum of the expected votes for parties z and $-z$ in both the primary and the general election is one then $\tilde{\phi}_0^z + \tilde{\phi}_0^{-z} = 1$, and $\phi_l^z + \phi_l^{-z} = 1\ \forall l$. Since $g_d^{*zi} = g_d^{*z,-i} = g_d^{*z}$ then $\frac{\partial \tilde{\phi}_0^{zi}}{\partial g_d^z} = \int_{\forall\, \tilde{e}^i} \tilde{h}^i(\tilde{e}^i)\frac{\partial \tilde{F}_0^{zi}}{\partial \tilde{\Psi}_0^{zi}}\frac{\partial \tilde{\Psi}_0^{zi}}{\partial g_d^z} de^i\ \forall i$ and $\frac{\partial \phi_l^{zi}}{\partial g_d^z} = \int_{\forall\, e^i} h^i(e^i)\frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}}\frac{\partial \Psi_l^{zi}}{\partial g_d^z} de^i$ for $l = 1,2, \forall i$. It follows that the first order condition is given by

$$\sum_{l=\{1,2\}} \frac{\partial \Pi_d^z}{\partial \rho_l^z}\left\{\sum_{\forall i,-i}\int_{\forall\, e^i} h^i(e^i)\frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}}\frac{\partial \Psi_l^{zi}}{\partial g_d^z} de^i\right\} = -\frac{\partial \Pi_d^z}{\partial \tilde{\rho}_0^z}\left\{\sum_{\forall i,-i}\int_{\forall\, \tilde{e}^i} \tilde{h}^i(\tilde{e}^i)\frac{\partial \tilde{F}_0^{zi}}{\partial \tilde{\Psi}_0^{zi}}\frac{\partial \tilde{\Psi}_0^{zi}}{\partial g_d^z} de^i\right\} \quad (A.11)$$

From the definition of the covariance between A and B, $\sigma(A,B) = E[AB] - E[A]E[B]$. Re-define $A = \left\{\frac{\partial \tilde{F}_0^{zi}}{\partial \tilde{\Psi}_0^{zi}}, \frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}}\right\}$ and $B = \left\{\frac{\partial \tilde{\Psi}_0^{zi}}{\partial g_d^z}, \frac{\partial \Psi_l^{zi}}{\partial g_d^z}\right\}$ for $l = 1,2$ to find:

$$\int_{\forall\, e^i} h^i(e^i)\frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}}\frac{\partial \Psi_l^{zi}}{\partial g_d^z} de^i =$$

$$\sigma_l^{zi}\left(\frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}}, \frac{\partial \Psi_l^{zi}}{\partial g_d^z}\right) + \left\{\int_{\forall\, e^i} h^i(e^i)\frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}} de^i\right\}\left\{\int_{\forall\, e^i} h^i(e^i)\frac{\partial \Psi_l^{zi}}{\partial g_d^z} de^i\right\} \qquad (A.12)$$

Define $\frac{\partial \tilde{\Psi}_0^{zi}}{\partial g_d^z} = \tilde{\upsilon}_g^{zi} = \frac{\partial \tilde{\mu}^{zi}}{\partial G_d^z} - \frac{\partial \tilde{\mu}^{zi}}{\partial \tilde{x}^{zi}}\left(\frac{1}{N}\right)$ and $\frac{\partial \Psi_l^{zi}}{\partial g_d^z} = \upsilon_g^{zi} = \frac{\partial \mu^{zi}}{\partial G^z} - \frac{\partial \mu^{zi}}{\partial x^{zi}}\left(\frac{1}{N}\right)$ $\forall l$ and use (A.12) into (A.11) to show

$$\sum_{\forall i,-i}\left\{\sum_{l=\{1,2\}}\frac{\partial \Pi_d^z}{\partial \rho_l^z}\int_{\forall e^i} h^i(e^i)\frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}}de^i \int_{\forall e^i} h^i(e^i)\frac{\partial \Psi_l^{zi}}{\partial g_d^z}de^i\right\} =$$

$$-\sum_{\forall i,-i}\sum_{l=\{1,2\}}\frac{\partial \Pi_d^z}{\partial \rho_l^z}\sigma_l^{zi}\left(\frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}},\frac{\partial \Psi_l^{zi}}{\partial g_d^z}\right) - \frac{\partial \Pi_d^z}{\partial \tilde{\rho}_0^z}\left\{\sum_{\forall i,-i}\int_{\forall \tilde{e}^i}\tilde{h}^i(\tilde{e}^i)\frac{\partial \tilde{F}_0^{zi}}{\partial \tilde{\Psi}_0^{zi}}\tilde{\upsilon}_g^{zi}de^i\right\} \quad (A.13)$$

Define $\alpha^{zi} \in (0,1): \sum_{\forall i,-i}\alpha^{zi} = 1$ where

$$\alpha^{zi} = \sum_{l=\{1,2\}}\frac{\partial \Pi_d^z}{\partial \rho_l^z}\int_{\forall e^i} h^i(e^i)\frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}}de^i \Big/ \sum_{\forall i,-i}\sum_{l=\{1,2\}}\frac{\partial \Pi_d^z}{\partial \rho_l^z}\int_{\forall e^i} h^i(e^i)\frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}}de^i \quad (A.14)$$

And

$$\gamma^z = \frac{\partial \Pi_d^z}{\partial \tilde{\rho}_0^z}\Big/ \sum_{\forall i,-i}\sum_{l=\{1,2\}}\frac{\partial \Pi_d^z}{\partial \rho_l^z}\int_{\forall e^i} h^i(e^i)\frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}}de^i \quad (A.15)$$

And

$$\sigma_\omega^z = \sum_{\forall i,-i}\sum_{l=\{1,2\}}\frac{\partial \Pi_d^z}{\partial \rho_l^z}\sigma_l^{zi}\left(\frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}},\frac{\partial \Psi_l^{zi}}{\partial g_d^z}\right)\Big/ \sum_{\forall i,-i}\sum_{l=\{1,2\}}\frac{\partial \Pi_d^z}{\partial \rho_l^z}\int_{\forall e^i} h^i(e^i)\frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}}de^i \quad (A.16)$$

Use (A.14), (A.15), and (A.16) into (A.13) to obtain the expression in (A.6)[2]

$$\sum_{\forall i,-i}\alpha^{zi}\int_{\forall e^i} h^i(e^i)\,\upsilon_g^{zi}(g_d^{*z})de^i$$

$$= -\gamma^z\left(\sum_{\forall i,-i}\int_{\forall \tilde{e}^i}\tilde{h}^i(\tilde{e}^i)\frac{\partial \tilde{F}_0^{zi}}{\partial \tilde{\Psi}_0^{zi}}\tilde{\upsilon}_g^{zi}(g_d^{*z})d\tilde{e}^i\right) - \sigma_\omega^z \quad (A.17)$$

[2] This is equivalent to expression (8) of Lemma 3 in the paper.

Lemma 4 *For economies with a decentralized party system and democratic decentralization, local public goods* g_{dL}^{*zi} $\forall\, i, -i$ *are provided such that* g_{dL}^{*zi} *satisfies the following:*

$$\int_{\forall\, e^i} h^i(e^i)\, \upsilon_{g_{dL}}^{zi}(g_{dL}^{*zi}) de^i = -\chi^{zi} \left(\int_{\forall\, \tilde{e}^i} \tilde{h}^i(\tilde{e}^i) \frac{\partial \tilde{F}_0^{zi}}{\partial \tilde{\Psi}_0^{zi}} \tilde{\upsilon}_{g_{dL}}^{zi}(g_{dL}^{*zi}) d\tilde{e}^i \right) - \sigma_\omega^{zi} \qquad (A.18)$$

Where

$$\chi^{zi} = \frac{\partial \pi_{dL}^{zi}}{\partial \tilde{\rho}_0^{zi}} \Bigg/ \sum_{l=\{1,2\}} \frac{\partial \pi_{dL}^{zi}}{\partial \rho_l^{zi}} \int_{\forall\, e^i} h^i(e^i) \frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}} de^i \qquad (A.19)$$

Where χ^{zi} *is a weighted rate of substitution between marginal changes in the party´s plurality in the district´s primary and the general local election, and*

$$\sigma_\omega^{zi} = \sum_{l=\{1,2\}} \frac{\partial \pi_{dL}^{zi}}{\partial \rho_l^{zi}}\, \sigma_l^{zi} \left(\frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}}, \frac{\partial \Psi_l^{zi}}{\partial g_{dL}^{zi}} \right) \Bigg/ \sum_{l=\{1,2\}} \frac{\partial \pi_{dL}^{zi}}{\partial \rho_l^{zi}} \int_{\forall\, e^i} h^i(e^i) \frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}} de^i \qquad (A.20)$$

Where σ_ω^{zi} *is a weighted covariance between the marginal probability that voter type* e^i *votes for party z in the local general election in district i,* $\frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}}$*, and the change in the well being of voters from an increase in the provision of the local public good* $\frac{\partial \Psi_l^{zi}}{\partial g_{dL}^{zi}}$.

Proof

In a federation with party decentralization, the spending policy of party z in district i is $g_{dL}^{*zi} \in argmax\ \pi_{dL}^{zi}\left(\tilde{\rho}_0^{zi}, \rho_l^{zi}\right) \forall\, l = \{1,2\}$.[3] The first order condition of the party´s problem is

$$\frac{\partial \pi_{dL}^{zi}}{\partial \tilde{\rho}_0^{zi}} \frac{\partial \tilde{\phi}_0^{zi}}{\partial g_{dL}^{zi}} + \sum_{l=\{1,2\}} \frac{\partial \pi_{dL}^{zi}}{\partial \rho_l^{zi}} \frac{\partial \phi_l^{zi}}{\partial g_{dL}^{zi}} = 0 \qquad (A.21)$$

From definition, $\tilde{\phi}_0^{zi} = \int_{\forall\, \tilde{e}^i} \tilde{h}^i(\tilde{e}^i)\, \tilde{F}_0^{zi}(\tilde{\Psi}_0^{zi}) d\tilde{e}^i$ and $\phi_l^{zi} = \int_{\forall\, e^i} h^i(e^i)\, F_l^{zi}(\Psi_l^{zi}) de^i$ for $l = \{1,2\}$. Obtain $\partial \tilde{\phi}_0^{zi} / \partial g_{dL}^{zi}$, $\partial \phi_l^{zi} / \partial g_{dL}^{zi}$ $\forall l$, and re-arrange terms to re-express (A.21) as follows

[3] Since the policies of candidates $j \in zi$ and $j' \in zi$ converge, we re-define $g_{dL}^{*jzi} = g_{dL}^{*j'zi} = g_{dL}^{*zi}$.

$$\sum_{l=\{1,2\}} \frac{\partial \pi_{dL}^{zi}}{\partial \rho_l^{zi}} \int_{\forall e^i} h^i(e^i) \frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}} \frac{\partial \Psi_l^{zi}}{\partial g_{dL}^{zi}} de^i = -\frac{\partial \pi_{dL}^{zi}}{\partial \tilde{\rho}_0^{zi}} \int_{\forall \tilde{e}^i} \tilde{h}^i(\tilde{e}^i) \frac{\partial \tilde{F}_0^{zi}}{\partial \tilde{\Psi}_0^{zi}} \frac{\partial \tilde{\Psi}_0^{zi}}{\partial g_{dL}^{zi}} de^i \qquad (A.22)$$

From the definition of the covariance between A and B, $\sigma(A,B) = E[AB] - E[A]E[B]$. Re-define $A = \left\{\frac{\partial \tilde{F}_0^{zi}}{\partial \tilde{\Psi}_0^{zi}}, \frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}}\right\}$ and $B = \left\{\frac{\partial \tilde{\Psi}_0^{zi}}{\partial g_{dL}^{zi}}, \frac{\partial \Psi_l^{zi}}{\partial g_{dL}^{zi}}\right\}$ for $l = 1,2$ to express the following

$$\int_{\forall e^i} h^i(e^i) \frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}} \frac{\partial \Psi_l^{zi}}{\partial g_{dL}^{zi}} de^i =$$

$$\sigma_l^{zi}\left(\frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}}, \frac{\partial \Psi_l^{zi}}{\partial g_{dL}^{zi}}\right) + \left\{\int_{\forall e^i} h^i(e^i) \frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}} de^i\right\}\left\{\int_{\forall e^i} h^i(e^i) \frac{\partial \Psi_l^{zi}}{\partial g_{dL}^{zi}} de^i\right\} \qquad (A.23)$$

Where $\sigma_l^{zi}\left(\frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}}, \frac{\partial \Psi_l^{zi}}{\partial g_{dL}^{zi}}\right)$ is the covariance between $\partial F_l^{zi}/\partial \Psi_l^{zi}$ and the voter´s marginal wellbeing, $\partial \Psi_l^{zi}/\partial g_{dL}^{zi}$, due to a change in the provision of the local public good in district i. Use conditions $\frac{\partial \tilde{\Psi}_0^{zi}}{\partial g_{dL}^{zi}} = \tilde{v}_{g_{dL}}^{zi} = \frac{\partial \tilde{\mu}^{zi}}{\partial G_{dL}^{zi}} - \frac{\partial \tilde{\mu}^{zi}}{\partial \tilde{x}^{zi}}\left(\frac{1}{N}\right)$ and $\frac{\partial \Psi_l^{zi}}{\partial g_{dL}^{zi}} = v_{g_{dL}}^{zi} = \frac{\partial \mu^{zi}}{\partial G_{dL}^{zi}} - \frac{\partial \mu^{zi}}{\partial x^{zi}}\left(\frac{1}{N}\right)\ \forall l$ and substitute (A.23) into (A.22) to establish

$$\int_{\forall e^i} h^i(e^i)\, v_{g_{dL}}^{zi} de^i = \frac{-\frac{\partial \pi_{dL}^{zi}}{\partial \tilde{\rho}_0^{zi}} \int_{\forall \tilde{e}^i} \tilde{h}^i(\tilde{e}^i) \frac{\partial \tilde{F}_0^{zi}}{\partial \tilde{\Psi}_0^{zi}} \frac{\partial \tilde{\Psi}_0^{zi}}{\partial g_{dL}^{zi}} de^i - \sum_{l=\{1,2\}} \frac{\partial \pi_{dL}^{zi}}{\partial \rho_l^{zi}} \sigma_l^{zi}}{\sum_{l=\{1,2\}} \frac{\partial \pi_{dL}^{zi}}{\partial \rho_l^{zi}} \int_{\forall e^i} h^i(e^i) \frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}} de^i} \qquad (A.24)$$

Define

$$\chi^{zi} = \frac{\partial \pi_{dL}^{zi}}{\partial \tilde{\rho}_0^{zi}} \Bigg/ \sum_{l=\{1,2\}} \frac{\partial \pi_{dL}^{zi}}{\partial \rho_l^{zi}} \int_{\forall e^i} h^i(e^i) \frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}} de^i \qquad (A.25)$$

And

$$\sigma_\omega^{zi} = \sum_{l=\{1,2\}} \frac{\partial \pi_{dL}^{zi}}{\partial \rho_l^{zi}}\ \sigma_l^{zi}\left(\frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}}, \frac{\partial \Psi_l^{zi}}{\partial g_{dL}^{zi}}\right) \Bigg/ \sum_{l=\{1,2\}} \frac{\partial \pi_{dL}^{zi}}{\partial \rho_l^{zi}} \int_{\forall e^i} h^i(e^i) \frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}} de^i \qquad (A.26)$$

To express condition (A.24) as follows

$$\int_{\forall e^i} h^i(e^i)\, v_{g_{dL}}^{zi}(g_{dL}^{*zi}) de^i = -\chi^{zi}\left(\int_{\forall \tilde{e}^i} \tilde{h}^i(\tilde{e}^i) \frac{\partial \tilde{F}_0^{zi}}{\partial \tilde{\Psi}_0^{zi}} \tilde{v}_{g_{dL}}^{zi}(g_{dL}^{*zi}) d\tilde{e}^i\right) - \sigma_\omega^{zi} \qquad (A.27)$$

Theorem 2 *In majoritarian democracies with a decentralized party system and open primaries, the strong decentralization theorem does not hold but the conventional decentralization theorem holds.*

Proof

For the economy with party decentralization and a system of local governments, the politically optimal policy for party z in each district is $g_{dL}^{*zi} > 0 \ \forall i, -i$ satisfying:

$$\int_{\forall e^i} h^i(e^i)\, \upsilon_g^{zi}(g_{dL}^{*zi}) de^i = -\chi^{zi} \left(\int_{\forall \tilde{e}^i} \tilde{h}^i(\tilde{e}^i) \frac{\partial \tilde{F}_0^{zi}}{\partial \widetilde{\Psi}_0^{zi}} \tilde{\upsilon}_g^{zi}(g_{dL}^{*zi}) d\tilde{e}^i \right) - \sigma_\omega^{zi} \qquad (A.28)$$

Under open primaries, the distribution of preferences over policy of voters voting in the primary is the same as the distribution of preferences of voters voting in the general election. Therefore, $h^i(e^i) = \tilde{h}^i(\tilde{e}^i)$ and $\upsilon_{g_{dL}}^{zi}(g_{dL}^{*zi}) = \tilde{\upsilon}_{g_{dL}}^{zi}(g_{dL}^{*zi})\ \forall z$. In this case, the parties' policies converge, therefore $\widetilde{\Psi}_0^{zi} = \widetilde{\Psi}_0^{-zi} = 0$, $\Psi_l^{zi} = \Psi_l^{-zi} = 0\ \forall z, \forall l$, $\frac{\partial \tilde{F}_0^{zi}(0)}{\partial \widetilde{\Psi}_0^{zi}} = c_0$ and $\frac{\partial F_l^{z}(0)}{\partial \Psi_l^{zi}} = c_1\ \forall z, \forall l$ where c_0, c_1 are positive constants. Hence $\sigma_l^{zi}\left(\frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}}, \frac{\partial \Psi_l^{zi}}{\partial g_{dL}^{zi}}\right) = 0\ \forall\ l \Rightarrow \sigma_\omega^{zi} = 0$ and $\int_{\forall \tilde{e}^i} \tilde{h}^i(\tilde{e}^i) \frac{\partial \tilde{F}_0^{zi}}{\partial \widetilde{\Psi}_0^{zi}} \tilde{\upsilon}_g^{zi}(g_{dL}^{*zi}) d\tilde{e}^i = \int_{\forall e^i} h^i(e^i)\, \tilde{\upsilon}_{g_{dL}}^{zi}(g_{dL}^{*zi}) de^i$. Moreover, $\tilde{\rho}_0^{zi} = 0$ and $\rho_l^{zi} = 0$ for $l = \{1,2\}$ leading to $\frac{\partial \pi_{dL}^{zi}(0)}{\partial \tilde{\rho}_0^{zi}} = \frac{\partial \pi_{dL}^{zi}(0)}{\partial \rho_l^{zi}} = c_2 \in \mathbb{R}_+\ \forall z$ and the following conditions hold:

$$\int_{\forall e^i} h^i(e^i) \frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}} de^i = c_1 \int_{\forall e^i} h^i(e^i)\, de^i = c_1 \in \mathbb{R}_+\ \forall z, \qquad (A.29)$$

Since $\int_{\forall e^i} h^i(e^i)\, de^i = 1$ measures the proportion of voters in the local election, and

$$\chi^{zi} = \frac{\partial \pi_{dL}^{zi}}{\partial \tilde{\rho}_0^{zi}} \Big/ \sum_{l=\{1,2\}} \frac{\partial \pi_{dL}^{zi}}{\partial \rho_l^{zi}} \int_{\forall e^i} h^i(e^i) \frac{\partial F_l^{zi}}{\partial \Psi_l^{zi}} de^i = c_2 / 2c_2 c_1 = constant \qquad (A.30)$$

Use (A.29) and (A.30) into (A.28) and re-arrange terms to show that the first order condition for $g_{dL}^{*zi}\ \forall i,-i$ satisfies the following condition:

$$\int_{\forall\, e^i} h^i(e^i)\, v_g^{zi}(g_{dL}^{*i})de^i = 0 \implies g_{dL}^{*zi} = g_{dL}^{*i}\ \ \forall\, z, \forall i \qquad (A.31)$$

Now consider an economy with party decentralization in open primaries and a nationwide government. By (A.4) in Lemma 3, the characterization of the first order condition for this economy with $g_d^{*z} > 0$ is given by:

$$\sum\nolimits_{\forall\, i,-i} \alpha^{zi} \int_{\forall\, e^i} h^i(e^i)\, v_g^{zi}(g_d^{*z})de^i$$
$$= -\gamma^z \left(\sum\nolimits_{\forall\, i,-i} \int_{\forall\, \tilde{e}^i} \tilde{h}^i(\tilde{e}^i) \frac{\partial \tilde{F}_0^{zi}}{\partial \tilde{\Psi}_0^{zi}} \tilde{v}_g^{zi}(g_d^{*z})d\tilde{e}^i \right) - \sigma_\omega^z \qquad (A.32)$$

In this equilibrium, the parties' policies also converge then $g_d^{*z} = g_d^*$ and following similar steps as those shown above imply that the first order condition in (A.32) can be expressed as follows:

$$\int_{\forall\, e^i} h^i(e^i)\, v_g^{zi}(g_d^*)de^i = -\int_{\forall\, e^{-i}} h^{-i}(e^{-i})\, v_g^{z,-i}(g_d^*)de^{-i} \qquad (A.33)$$

Moreover, recall from condition (2) that $\hat{g}^{*i} > 0\ \forall\, i$: $\hat{g}^{*i} \in \arg\max \int_{\forall\, e^i} h^i(e^i)\, v^i(e^i, G^i)de^i$ is the global maximizer of the aggregate well-being of residents of district i such that it is satisfied[4]

$$\int_{\forall\, e^i} h^i(e^i)\, v_g^i(\ \hat{g}^{*i})de^i = 0 \quad \forall i\ \ \hat{g}^{*i} > 0 \qquad (A.34)$$

Therefore

$$\int_{\forall\, e^i} h^i(e^i)\, v^i(\hat{g}^{*i})de^i \ \geq \int_{\forall\, e^i} h^i(e^i)\, v^i(g^i)de^i \quad \forall\, g^i \neq \hat{g}^{*i} \qquad (A.35)$$

Without loss of generality, the heterogeneity of preferences of voters for public goods means $g^{*i} \geq g^{*-i}$.[5] Moreover, condition (A.31) of Theorem 2 and condition (2) of proposition 1 imply

[4] See that condition (A.34) is equivalent to condition (2).

[5] The heterogeneity of preferences means that, in general, $g^{*i} \geq g^{*-i}$ or $g^{*i} \leq g^{*-i}$. For the purpose of our analysis, and without loss of generality, we assume $g^{*i} \geq g^{*-i}$.

that $g_{dL}^{*i} = \hat{g}^{*i}\ \forall i$, and the expressions in (A.33) and (A.34) imply $g_{dL}^{*-i} \leq g_d^* \leq g_{dL}^{*i}$. These outcomes and condition (A.35) means

$$\int_{\forall\, e^i} h^i(e^i)\, v^i(g_{dL}^{*i}) de^i + \int_{\forall\, e^{-i}} h^{-i}(e^{-i})\, v^{-i}(g_{dL}^{*-i}) de^{-i} \geq \sum\nolimits_{\forall i,-i} \int_{\forall\, e^i} h^i(e^i)\, v^i(g_d^*) de^i \quad (A.36)$$

■

Theorem 3 *The strong and the conventional decentralization theorems do not hold in majoritarian democracies with decentralized party systems and closed primaries.*

Proof

A party z seeking to form a central government in politically decentralized regimes with closed primaries selects $g_d^{*z} \in argmax\, \Pi_d^z(\tilde{\rho}_0^z, \rho_1^z, \rho_2^z)$ subject to $g_d^{*zi} = g_d^{*z,-i} = g_d^{*z}$. Moreover $\hat{g}^{*i} \in \arg\max \int_{\forall\, e^i} h^i(e^i)\, v^i(e^i, G^i) de^i\ \forall i$ where $\hat{g}^{*i}$ is the policy that maximizes the nationwide surplus from the fiscal exchange associated with a local public good in district i. Condition (A.6) of Lemma 3 and condition (2) from proposition 1 imply that, in general, $g_d^{*z} \neq \hat{g}^{*i}\ \forall i$. Similarly, in a system of local governments with party decentralization and closed primaries, party z selects $g_{dL}^{*zi} \in argmax\ \pi_{dL}^{zi}\ \forall i$. Condition (A.18) from Lemma 4 and condition (2) from proposition 2 show that, in general, $g_{dL}^{*zi} \neq \hat{g}^{*i}\ \forall i$. As a result, in general, the nationwide aggregate wellbeing of voters satisfies the following

$$\int_{\forall\, e^i} h^i(e^i)\, v^i(g_{dL}^{*zi}) de^i + \int_{\forall\, e^{-i}} h^{-i}(e^{-i})\, v^{-i}(g_{dL}^{*z,-i}) de^{-i} \lesseqgtr$$

$$\sum\nolimits_{\forall i,-i} \int_{\forall\, e^i} h^i(e^i)\, v^i(g_d^{*z}) de^i \quad (A.37)$$

■