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The Provision of Local Public Goods in Proportional Representation Electoral Systems with Closed and Open Party Lists.

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Abstract

In this paper we find that the institutional set up of proportional representation systems matters for the welfare properties related with the ideal structure of government providing local public goods. In particular, we study the role of party centralization vs party decentralization in the provision of local public goods. In this paper, we that the provision of local public goods *with inter-regional spillovers* by a system of local governments (welfare) dominates the fiscally centralized provision in economies with a proportional representation electoral system and closed party lists. We call this outcome the strong decentralization theorem. For this type of economies, the conventional decentralization theorem (originally identified by Oates 1972) is also satisfied. For economies with a proportional representation and open party lists systems the strong decentralization theorem is satisfied only when party centralization (i.e., the ability of party leaders to nominate candidates in the party's lists) plays a dominant role in determining the policy platforms of candidates. However, if there is party decentralization (parties lack the ability to influence policy through the nomination process in the party's list) the strong decentralization theorem is not satisfied. Lastly, the conventional decentralization theorem is satisfied in economies with proportional representation electoral systems and open party lists in both type of party systems: centralized and decentralized.

JEL: D61, D72, D78, H73, H75, H77
Keywords: Allocative Efficiency, Elections, local Public Goods, Fiscal Federalism.

I Introduction

The analysis of the optimal structure of government to serve citizens is at the core of the theory of fiscal federalism. In his seminal work, Oates (1972, p. 54) identifies conditions for the well known decentralization theorem: ". . . in the absence of cost-savings from the centralized provision of a (local public) good and of inter-jurisdictional externalities, the level of welfare will always be at least as high (and typically higher) if Pareto-efficient levels of consumption are provided in each jurisdiction than if any single, uniform level of consumption is maintained across all jurisdictions". The analysis of Oates (1972) shows a tradeoff between the heterogeneity of preferences and efficiency for the optimal structure of government to provide local public goods. The analysis of Oates (1972) implies that, with heterogeneous districts, a system of local governments (welfare) dominates a central government when there is no spillovers and a central government dominates a federation when districts are homogeneous and there is spillovers of local public goods.

In the analysis of Oates (1972) the government is ruled by benevolent social planners. In contrast, several economists have emphasized the importance of considering the role of political processes and institutions, such as elections and legislatures, that shape the fiscal choices made by policy makers. However, there is little systematic theory on the type of political institutions that might lead to the welfare gains expected from the decentralization of public spending to a system of local governments. Because the decentralization theorem ignores some political realities, it is not clear how party systems, the structure of the electoral system, and the electoral competition influence the parties' fiscal choices and, ultimately, the welfare properties of a fiscally decentralized versus decentralized provision of local public goods.

In this paper we contribute in filling this gap by analyzing the role of the centralization and decentralization of parties in democracies with proportional representation (PR) electoral systems on the optimal structure of government to serve their citizens. Empirical evidence shows that there are democracies with party centralized regimes. In this case, party leaders have significant control over the nomination of candidates of their parties and the design of the parties' policy platforms (see Riker 1964).

In contrast, there are democracies with decentralized party systems in which the nomination of their candidates is decided by the electorate. Proponents of decentralized party systems argue that party centralization induces politicians to adopt policy platforms that please party leaders since they control the mechanism of the nomination of candidates while in decentralized party systems the nomination of candidates is determined by voters. Therefore, party decentralized regimes could promote the political participation of voters, the representation of their interests in the policies implemented by the government, and the accountability of public officials.

In this paper we are interested in studying the role of party (de)centralization on the expected benefits of fiscal decentralization in economies with a proportional representation electoral system with closed and open party list systems. The case of party centralization is characterized by a proportional representation electoral system with closed party lists and the case for party decentralization by an electoral system with open party lists. In a PR system with closed party lists, party leaders have full control of the nomination process and determine not only the names of candidates to appear in the list but also the candidates' ranking on the list (and therefore the candidates' chance to get a seat in the legislature).

In a PR system with open party lists, party leaders determine the candidates to appear on the list but the ranking of candidates is determined by the electorate (with the candidate receiving the highest number of votes in the first position of the list, the candidate with the second largest number of votes appear in the second place, and so on). The structure of this electoral system creates incentives for candidates to recognize that they not only need to please party leaders but also need to get votes from the electorate to secure a high position in the party's and a seat in the legislature.

For our economy, the design of policy is analyzed through a four stage electoral-legislative process: in the first stage of the game parties announce policy platforms and the list of candidates. In the second stage of the electoral process, individuals vote for the party that advances the policy platform that is closest to their own views on public spending. In the third stage, the election takes place and parties receive a proportion of the seats in the legislature equivalent to the proportion of votes received in the election. In the fourth stage, the legislative bargaining takes place and a policy is implemented. Finally, we develop a comparative analysis of the welfare properties of the provision of local public goods when there is fiscal centralization and decentralization for economies with a PR electoral system and with closed and open party lists.

This paper contributes to the analysis of the role of political institutions and the theory of fiscal federalism in several ways: First, we provide a stronger case for fiscal decentralization relative the decentralization theorem identified by Oates (1972). In particular, we show that a system of local governments is welfare superior to the centralized provision of local public goods with spillovers for a democracy with a proportional representation electoral system with closed party lists. We call this result the strong decentralization theorem. We also show that the conventional decentralization theorem holds for an economy with a proportional representation electoral system and closed party lists.

Second, for economies with a proportional representation and open party lists systems the strong decentralization theorem is satisfied only when party centralization (i.e., the ability of party leaders to nominate candidates in the party's lists) plays a dominant role in determining policy for candidates. However, if there is party decentralization (parties lack the ability to influence policy through the nomination process in the party's list) the strong decentralization theorem is not satisfied. Lastly, the conventional decentralization theorem is satisfied is economies with proportional representation electoral systems and open party lists in both type of party systems: centralized and decentralized.

The paper is structured as follows. Section II introduces our model of the provision of local public goods by a central government for an economy with a proportional representation electoral system with closed party lists. The analysis for a fiscally decentralized provision and the comparative analysis of the welfare properties of fiscal (de)centralization is presented in section III. Section IV considers an economy with a PR electoral system with open party lists and fiscal centralization. Section V provides the analysis for a fiscally decentralized provision of local public goods and the comparative analysis of the welfare properties of fiscal (de)centralization for an economy with a PR electoral system and open party lists. Section VI concludes.

II Proportional Representation with a Closed Party List System and Fiscal Centralization

In this section we incorporate the analysis of the provision of local public goods (LPG's) in proportional representation (PR) systems with closed party lists. There are several distinctions of this section with the model of the provision of local public goods in a majoritarian electoral system of Ponce, Hankla, Martinez and Heredia (2012). First, in this economy we will consider electoral competition among multiple (three) parties. Second, our political equilibrium incorporates an electoral process in which candidates and parties compete for votes and a election takes place and a legislative process in which parties negotiate public policy. In this paper we will study the differences on the fiscally (de)centralized provision of local public goods in democracies with proportional representation systems and closed and open party lists.

In our economy, three parties, labeled $z = L, M, R$, compete in the election to form the government. Under a central government, local public goods are provided by a single government that represents voters of all districts. The government finances its expenditures through a uniform tax on residents of all districts. We follow the literature by assuming that local public goods provided by the central government are uniform across districts (see Oates 1972, 1995).

Following Ponce, Hankla, Martinez and Heredia (2012) we consider an economy constituted by districts i and $-i$ with $n^i = 1,2 \ldots \ldots N$ individuals in each district. Individuals do not have mobility across jurisdictions. The preferences of an individual with an endowment e^i in district i is $v^i(e^i, G^i) = Max\, \mu^i(x^i, g^i, g^{-i}) = x^i G^i$ subject to $a)$ $x^i = e^i - t^i$ and $b)$ $g^i = Nt^i$ $\forall i$, where $v^i(e^i, G^i)$ is the individual´s indirect utility, $\mu^i(x^i, g^i, g^{-i})$ are his preferences over a private good x^i, $G^i = \left(g^i + k^{-i}g^{-i}\right)$ is the overall consumption of local public goods provided by district i, g^i, and by district $-i$, g^{-i}, and t^i is a head tax on residents of district i

The parameter $k^{-i} \in [0, 1)\forall - i, i,$ measures the extent of inter-regional spillovers of g^{-i} over residents of district i. For local public goods without spillovers $k^{-i} = 0 \ \forall - i, i,$ and $k^{-i} = 1$ when local spending in district $-i$ is over a nationwide pure public good. Condition (a) is the individual's budget constraint. The distribution of heterogeneous endowments across districts is given by $e^i \in \left[\underline{e^i}, \bar{e}^i\right]: h^i\left(e^i\right) > 0 \quad \forall i$ with $\sum_{\forall i,-i} \int_{\forall e^i} h^i\left(e^i\right) de^i = 1.$ Condition (b), $g^i = Nt^i \ \forall i,$ is the constraint that public goods are financed by taxes.[1]

In the first stage of the game parties announce policy platforms and the list of candidates. During the second stage of the electoral process, individuals vote for the party that advances the spending policy that is closest to their own views on public spending. In the third stage, the election takes place and parties receive a proportion of the seats in the legislature equivalent to the proportion of votes received in the election. In the fourth stage, the legislative bargaining takes place. We assume an exogenously given policy $\mathbf{g_0} = \left[g_0^i, g_0^{-i}\right]$ as the status quo where g_0^i is the size of public spending in district i and a similar interpretation is given to g_0^{-i}. We follow Austen-Smith (2000) in considering the next legislative procedure: The party with the highest share of the vote proposes a policy. If this party has a majority of the seats this policy is implemented. If no party has a majority of the seats in the legislature then one party is selected

[1] The government´s budget constraints say that g^i is financed by a head tax applied only to residents of the district. This configuration allows us to eliminate any possible gains of economies of scale in the provision of local public goods by the central government over sub-national governments. We impose this condition to evaluate whether the Decentralization Theorem of Oates (1972) holds in modern democracies once we introduce political institutions and incentives instead of governments controlled by benevolent social planners.

randomly and proposes a policy. If this policy receives a majority of the votes then the policy is implemented. If the policy does not receive the majority then the status quo is implemented.

In the second stage of the electoral process voters observe the parties' policies and vote for the policy that is closest to the voter's ideal policy on public spending. We assume that voters only care about policy and that the characteristics of candidates do not affect voting behavior.[2] To characterize voting behavior, assume $z = L$ and denote $\Psi_c^{zMi} = v^{zi}(e^i, g_c^{zi}) - v^{Mi}(e^i, g_c^{Mi})$ where Ψ_c^{zMi} is the difference in the voter's payoff with endowment e^i in district i if party z is elected and implements policies g_c^{zi} and $g_c^{z,-i}$ in districts i and $-i$ instead of the alternative policies g_c^{Mi} and $g_c^{M,-i}$ when party M is elected. Similarly, consider $\Psi_c^{zRi} = v^{zi}(e^i, g_c^{zi}) - v^{Ri}(e^i, g_c^{Ri})$. The voter type e^i votes for party z if $\Psi_c^{ZMi} > 0$ and $\Psi_c^{ZRi} > 0$. We assume voting is sincere and sequentially rational.

In a closed party list system, party centralization reflects the ability of party leaders to nominate candidates that can credibly commit to the policy platform adopted by the party. It should be obvious that, under party centralization, to nominate candidates to hold seats in the legislature that are not going to carry out the party's policy platform is a dominanted strategy. The list with the names of candidates is characterized by the vector $\mathbf{\Theta^z} = [\Theta^{zi}, \Theta^{z,-i}]$ where $\Theta^{zi} = \{N_j^{zi}\}_{j=1}^{J}$ is a sequence N_j^{zi} reflecting the name of candidate of party z in district i with order $j = 1,2 \ldots J$. Thus, if a party receives three seats in the legislature the names of candidates $N_1^{zi}, N_2^{zi}, N_3^{zi}$ occupy those seats in the legislature. We assume that parties have a supply of candidates given by $J' \geq J$.

[2] This assumption is for simplicity of the analysis. However, in the future we will relax this assumption.

From the point of view of parties, the individual's vote is uncertain (voting is probabilistic). The probability that a voter type e^i votes for party z in district i is $F_c^{zi}(\Psi_c^{zMi}, \Psi_c^{zRi}) = \int_{-\infty}^{\Psi_c^{zMi}} \int_{-\infty}^{\Psi_c^{zRi}} f_c^{zi}(\Psi_c^{zMi}, \Psi_c^{zRi})\, d\Psi_c^{zMi} d\Psi_c^{zRi}$, where $f_c^{zi}(\Psi_c^{zMi}, \Psi_c^{zRi})$ is a continuous probability distribution over Ψ_c^{zMi} and Ψ_c^{zRi}. The expected proportion of the vote of party z in district i is $\phi_c^{zi} = \int_{\forall\, e^i} h^i(e^i)\, F_c^{zi}(\Psi_c^{zMi}, \Psi_c^{zRi}) de^i$ and the expected proportion of the vote in both districts is $\phi_c^z = \sum_{\forall i,-i} \phi_c^{zi}$.[3]

Formally the sub-game perfect Nash equilibrium of this electoral-legislative game is characterized by proposition 1.

Proposition 1. *The sub-game perfect Nash equilibrium for the fiscally centralized provision of local public goods in economies with PR electoral systems and closed party lists is given by:*

1.i) In the first stage of the game party z proposes the policy platform $\boldsymbol{g}_c^{**\mathbf{z}} = [g_c^{**zi}, g_c^{**z,-i}]$ *and a list of candidates* $\boldsymbol{\Theta}^{*\mathbf{z}} = [\Theta^{*zi}, \Theta^{*z,-i}]$ *who agree with this policy such that*

$$\mathbf{g}_\mathbf{c}^{**\mathbf{z}} \in argmax \;\; \phi_c^z = \sum_{\forall i,-i} \int_{\forall\, e^i} h^i(e^i)\, F_c^{zi}(\Psi_c^{zMi}, \Psi_c^{zRi}) de^i$$

$$subject \;\; to \;\; g_c^{zi} = g_c^{z,-i} = g_c^z \;\; \forall\, z \qquad (1)$$

1.ii) In the second stage of the game, a voter type e^i *in district i votes for party z if*

$$\begin{aligned} \Psi_c^{zMi} &= v^{zi}(e^i, g_c^{***zi}) - v^{Mi}(e^i, g_c^{***Mi}) > 0 \\ \Psi_c^{zRi} &= v^{zi}(e^i, g_c^{***zi}) - v^{Ri}(e^i, g_c^{***Ri}) > 0 \end{aligned} \qquad (2)$$

[3] Our notation $\sum_{\forall i,-i} \phi_c^{zi}$ means that $\sum_{\forall i,-i} \phi_c^{zi} = \phi_c^{zi} + \phi_c^{z,-i}$.

Where $\boldsymbol{g_c^{***z}} = \left[g_c^{***zi}, g_c^{***z,-i}\right]$ *is the policy to be implemented in the legislative process (in the fourth stage) while policy* $\boldsymbol{g_c^{**z}} = \left[g_c^{**zi}, g_c^{**z,-i}\right]$ *corresponds to the policy platform of party z.*[4] *Otherwise the voter votes for party* M *or party* R.

1.iii) In the third stage of the game, votes are counted and the shares of the votes and seats are determined.

1.iv) In the fourth stage the status quo policy is given by $\mathbf{g_0} = \left[g_0^i, g_0^{-i}\right]$ *and the legislative bargaining game takes place as follows:*

a. *If* $\phi_c^z > \frac{1}{2}$ *for some party z then* $\boldsymbol{g_c^{***z}} = \mathbf{g_c^{**z}} = \left[g_c^{**zi}, g_c^{**z,-i}\right]$ *is proposed and approved by majority.*
b. *If* $\phi_c^z < \frac{1}{2}\ \forall\ z = \{L, M, R\}$ *then randomly a party* $z = \{L, M, R\}$ *proposes:*
    - *A policy* $g_c^{***zi} = g_0^i$ *if* $\left|g_0^i - g_c^{***zi}\right| < \left|g_{MV}^i - g_c^{***zi}\right|\ \forall i, -i$ *where* g_{MV}^i *is the median voter policy in district i.*
    - *Otherwise party z proposes a policy* $g_c^{***zi} = g_{MV}^i$ *if* $\left|g_0^i - g_c^{***zi}\right| \geq \left|g_{MV}^i - g_c^{***zi}\right|$

Proposition 1 characterizes weakly dominant strategies for parties, candidates and voters at the electoral and legislative stages. At the first stage, any party z proposes a list of candidates that endorses a policy platform that maximizes the party's share of the vote in the election. In a

[4] Note that sequential rationality requires voters to vote for the policy to be determined in the legislative stage, that is $\mathbf{g_c^{***z}} = \left[g_c^{***zi}, g_c^{***z,-i}\right]$ instead of the policy platform given by the vector $\mathbf{g_c^{**z}} = \left[g_c^{**zi}, g_c^{**z,-i}\right]$.

closed party list system, party centralization reflects the ability of party leaders to nominate candidates to hold seats in the legislature that are going to carry out the party's policy platform. This policy is the policy that maximizes the party's nationwide share of the vote. Voters vote sincerely not for the policy of the electoral stage $\mathbf{g}_{\mathbf{c}}^{**\mathbf{z}} = \left[g_c^{**zi}, g_c^{**z,-i}\right]$ but for the policy to be implemented at the legislative stage $\mathbf{g}_{\mathbf{c}}^{***\mathbf{z}} = \left[g_c^{***zi}, g_c^{***z,-i}\right]$.

At the legislative game, parties take into account the distribution of seats and the distribution of ideal policies of all parties $z = \{L, M, R\}$. Proposition 1 says that if a party has a majority of the seats then the ideal electoral platform policy $\mathbf{g}_{\mathbf{c}}^{***\mathbf{z}} = \mathbf{g}_{\mathbf{c}}^{**\mathbf{z}} = \left[g_c^{**zi}, g_c^{**z,-i}\right]$ is proposed and approved by majority**.** Otherwise, the party will propose a policy $\mathbf{g}_{\mathbf{c}}^{***\mathbf{z}}$ that is closest to the ideal electoral platform of the party ($\mathbf{g}_{\mathbf{c}}^{**\mathbf{z}}$**).** The policy to be implemented in the last stage of the game is characterized by condition *1.iv.b*.

The electoral platform on local public goods for a democracy with a PR system with a closed party list and a nationwide election to form the central government, $\mathbf{g}_{\mathbf{c}}^{**\mathbf{z}} = \left[g_c^{**zi}, g_c^{**z,-i}\right]$, is characterized in proposition 2.

Proposition 2. *Parties* $z = \{L, M, R\}$ *converge in selecting* $\mathbf{g}_{\mathbf{c}}^{**\mathbf{z}} = \left[g_c^{**zi}, g_c^{**z,-i}\right]$, $\forall z$ *for an economy with a PR system with closed party lists. Therefore at the legislative stage the policy* $\mathbf{g}_{\mathbf{c}}^{***\mathbf{z}} = \mathbf{g}_{\mathbf{c}}^{**\mathbf{z}}$ *is implemented by unanimity.*

Proof

The function of the share of the vote $\phi_c^z = \phi_c^z\left(\Psi_c^{zMi}, \Psi_c^{zRi}\right)$ is continuous and strictly concave. Hence

$$\mathbf{g_c^{**z}} \in argmax \;\; \phi_c^z = \sum_{\forall i,-i} \int_{\forall\, e^i} h^i(e^i)\, F_c^{zi}(\Psi_c^{zMi}, \Psi_c^{zRi}) de^i$$

$$st\!: \; g_c^{zi} = g_c^{z,-i} = g_c^z \;\; \forall\, z \tag{3}$$

Define $\mathbf{g_c^{**z}} = [g_c^{**zi}, g_c^{**z,-i}]$, $\forall z$, ξ_c^z and $\delta_{CL,c}^z(\mathbf{g_c^z}, \xi_c^z) = \phi_c^z + \xi_c^z\{g_c^{zi} - g_c^{z,-i}\}$ where ξ_c^z is a Lagrange multiplier. The strict concavity of ϕ_c^z implies that the Hessian matrix of $\delta_{CL,c}^z(\mathbf{g_c^z}, \xi_c^z)$ $\mathbf{H}\left(\delta_{CL,c}^z(\mathbf{g_c^z}, \xi_c^z)\right)$ is negative definite. For the case $\mathbf{g_c^{**z}}$, ξ_c^{**z} satisfies $\partial\delta_c/\partial g_c^{zi} = 0 \;\; \forall g_c^{**zi} > 0$ and $\partial\delta_c/\partial\xi_c^z = 0 \; \forall \; \xi_c^{**z} \neq 0$ then $\mathbf{g_c^{**z}}$ is a global maximizer of ϕ_c^z on the constrained policy set. Hence $\mathbf{g_c^{**z}}$ is unique which implies parties converge in selecting policy platforms $\mathbf{g_c^{**L}} = \mathbf{g_c^{**M}} = \mathbf{g_c^{**R}}$ and this policy is approved by unanimity at the legislative stage.

■

Proposition 2 says that the parties' share of the vote is a continuous and strictly concave function. The strict concavity of ϕ_c^z imply that the first order conditions are sufficient and necessary for an optimum of the parties' problem of policy design. Moreover, the strict concavity of ϕ_c^z also implies that the parties' solution is unique therefore the parties' policies converge. The convergence of the parties' policies also implies that a random party $z = \{L, M, R\}$ proposes its ideal policy platform $\mathbf{g_c^{**z}}$ which is approved by unanimity in the legislature in the fourth stage of our electoral-legislative game.

Lemma 1 *Local public goods with and without spillovers are Pareto efficient for an economy with a PR electoral system with closed party lists and a fiscally centralized government. All* *parties converge in providing a uniform local public good across districts,* $g_c^{**zi} = g_c^{**z,-i} = g_c^{**}\ \ \forall\, z = \{L, M, R\}$ *satisfying*

$$\sum_{\forall\, i,-i}\left(1+k^{-i}\right)\int_{\forall\, e^i} h^i\!\left(e^i\right)\frac{\partial \mu^i}{\partial G^i}\,de^i = \left\{\frac{1}{N}\right\}\sum_{\forall i,-\,i}\int_{\forall\, e^i} h^i\!\left(e^i\right)\frac{\partial \mu^i}{\partial x^i}\,de^i = 0 \qquad (4)$$

Proof

The parties' problem is $Max\ \delta_{CL,c}^{z}(\mathbf{g_c^z}, \xi_c^z\,) = \phi_c^z + \xi_c^z\{g_c^{zi} - g_c^{z,-i}\}$. The first order condition for $g_c^{**zi} > 0$ is

$$\int_{\forall\, e^i} h^i\!\left(e^i\right)\left\{\frac{\partial F_c^{zi}}{\partial \Psi_c^{zMi}}\frac{\partial \Psi_c^{zMi}}{\partial g_c^{**zi}} + \frac{\partial F_c^{zi}}{\partial \Psi_c^{zRi}}\frac{\partial \Psi_c^{zRi}}{\partial g_c^{**zi}}\right\}de^i$$

$$+\int_{\forall\, e^{-i}} h^{-i}\!\left(e^{-i}\right)\left\{\frac{\partial F_c^{z,-i}}{\partial \Psi_c^{zM,-i}}\frac{\partial \Psi_c^{zM,-i}}{\partial g_c^{**zi}} + \frac{\partial F_c^{z,-i}}{\partial \Psi_c^{zR,-i}}\frac{\partial \Psi_c^{zR,-i}}{\partial g_c^{**zi}}\right\}de^i + \xi_c^z = 0\ \ \ \forall g_c^{**zi} > 0 \qquad (5)$$

And

$$\int_{\forall\, e^i} h^i\!\left(e^i\right)\left\{\frac{\partial F_c^{zi}}{\partial \Psi_c^{zMi}}\frac{\partial \Psi_c^{zMi}}{\partial g_c^{**z,-i}} + \frac{\partial F_c^{zi}}{\partial \Psi_c^{zRi}}\frac{\partial \Psi_c^{zRi}}{\partial g_c^{**z,-i}}\right\}de^i$$

$$+\int_{\forall\, e^{-i}} h^{-i}\!\left(e^{-i}\right)\left\{\frac{\partial F_c^{z,-i}}{\partial \Psi_c^{zM,-i}}\frac{\partial \Psi_c^{zM,-i}}{\partial g_c^{**z,-i}} + \frac{\partial F_c^{z,-i}}{\partial \Psi_c^{zR,-i}}\frac{\partial \Psi_c^{zR,-i}}{\partial g_c^{**z,-i}}\right\}de^{-i} - \xi_c^z = 0\ \ \ \forall g_c^{**z,-i} \qquad (5')$$

Since $\frac{\partial \Psi_c^{zMi}}{\partial g_c^{**zi}} = \frac{\partial \Psi_c^{zRi}}{\partial g_c^{**zi}} = \frac{\partial \upsilon^{zi}}{\partial g_c^{**zi}}$ and $\frac{\partial \Psi_c^{zM,-i}}{\partial g_c^{**zi}} = \frac{\partial \Psi_c^{zR,-i}}{\partial g_c^{**zi}} = \frac{\partial \upsilon^{z,-i}}{\partial g_c^{**zi}}$ then conditions (5) and (5') imply

$$\int_{\forall\, e^i} h^i\!\left(e^i\right)\left\{\frac{\partial F_c^{zi}}{\partial \Psi_c^{zMi}} + \frac{\partial F_c^{zi}}{\partial \Psi_c^{zRi}}\right\}\frac{\partial \upsilon^{zi}}{\partial g_c^{**zi}}\,de^i + \int_{\forall\, e^{-i}} h^{-i}\!\left(e^{-i}\right)\left\{\frac{\partial F_c^{z,-i}}{\partial \Psi_c^{zMi}} + \frac{\partial F_c^{z,-i}}{\partial \Psi_c^{zRi}}\right\}\frac{\partial \upsilon^{z,-i}}{\partial g_c^{**zi}}\,de^{-i} =$$

$$\int_{\forall\, e^i} h^i\!\left(e^i\right)\left\{\frac{\partial F_c^{zi}}{\partial \Psi_c^{zMi}} + \frac{\partial F_c^{zi}}{\partial \Psi_c^{zRi}}\right\}\frac{\partial \upsilon^{zi}}{\partial g_c^{**z,-i}}\,de^i + \int_{\forall\, e^{-i}} h^{-i}\!\left(e^{-i}\right)\left\{\frac{\partial F_c^{z,-i}}{\partial \Psi_c^{zMi}} + \frac{\partial F_c^{z,-i}}{\partial \Psi_c^{zRi}}\right\}\frac{\partial \upsilon^{z,-i}}{\partial g_c^{**z,-i}}\,de^{-i} \quad (6)$$

Because the parties' policies converge then $\frac{\partial F_c^{zi}}{\partial \Psi_c^{zMi}} = \frac{\partial F_c^{zi}}{\partial \Psi_c^{zRi}} = \frac{\partial F_c^{z,-i}}{\partial \Psi_c^{zM,-i}} = \frac{\partial F_c^{z,-i}}{\partial \Psi_c^{zR,-i}} = f_c^{zi}(0) = f_c^{z,-i}(0) \in \mathbb{R}_+\ \ \forall\, i, \forall\, z$. Therefore the first order condition becomes

$$\int_{\forall\, e^i} h^i(e^i)\frac{\partial v^{zi}}{\partial g_c^{**zi}}de^i + \int_{\forall\, e^{-i}} h^{-i}(e^{-i})\frac{\partial v^{z,-i}}{\partial g_c^{**zi}}de^i$$

$$+\int_{\forall\, e^i} h^i(e^i)\frac{\partial v^{zi}}{\partial g_c^{**z,-i}}de^i + \int_{\forall\, e^{-i}} h^{-i}(e^{-i})\frac{\partial v^{z,-i}}{\partial g_c^{**z,-i}}de^i = 0 \qquad (7)$$

Use $\frac{\partial v^{zi}}{\partial g_c^{**zi}} = \frac{\partial \mu^i}{\partial G^i} - \left(\frac{1}{N}\right)\frac{\partial \mu^i}{\partial x^i}$, $\frac{\partial v^{z,-i}}{\partial g_c^{**zi}} = k^{-i}\frac{\partial \mu^{-i}}{\partial G^{-i}}$, $\frac{\partial v^{zi}}{\partial g_c^{**z,-i}} = k^i\frac{\partial \mu^i}{\partial G^i}$, and $\frac{\partial v^{z,-i}}{\partial g_c^{**zi}} = \frac{\partial \mu^{-i}}{\partial G^{-i}} - \left(\frac{1}{N}\right)\frac{\partial \mu^{-i}}{\partial x^{-i}}$ to show that the uniform local public good g_c^* satisfies

$$\sum_{\forall\, i,-i}(1+k^{-i})\int_{\forall\, e^i} h^i(e^i)\frac{\partial \mu^i}{\partial G^i}de^i = \left\{\frac{1}{N}\right\}\sum_{\forall i,-\,i}\int_{\forall\, e^i} h^i(e^i)\frac{\partial \mu^i}{\partial x^i}de^i \quad \forall g_c^{**zi} > 0 \qquad (8)$$

■

Lemma 1 characterizes Pareto efficient local public goods with and without spillovers for economies with PR electoral systems and closed party lists. In these economies, parties have electoral incentives to recognize the nationwide distribution of benefits of local public goods, i.e, the marginal benefits of g_c^{**zi} of residents of district i, $\int_{\forall\, e^i} h^i(e^i)\frac{\partial \mu^i}{\partial G^i}de^i$, the spillover effects of g_c^{**zi} on district $-i$, $k^{-i}\int_{\forall\, e^{-i}} h^{-i}(e^{-i})\frac{\partial \mu^{-i}}{\partial G^{-i}}de^{-i}$, against the marginal electoral costs of g_c^{**zi} $\int_{\forall\, e^i} h^i(e^i)\frac{\partial \mu^i}{\partial x^i}de^i$ which are the costs of forgone private consumption of residents of district i.[5]

The constraint that local public goods must be uniform across districts imply that the net marginal benefits from the public good in district i must be the same to the net marginal benefits

[5] The financing of local public goods require the payment of taxes of residents of district i. These taxes imply a fall in income which in turn leads reduces private consumption of residents of district i.

from the public good in district $-i$. This equivalence of the net marginal benefits across districts leads to condition (8).[6]

III Proportional Representation with Closed Party Lists and Fiscal Decentralization

In this section, we extend our analysis of the provision of local pubic goods for an economy with a PR system and closed party lists when the economy is fiscally decentralized. In this case there are two local elections in districts i and $-i$. Local public goods are provided by a system of local autonomous governments that represent voters of districts i and $-i$. A local government of district i finances its expenditures through a uniform tax on residents of the district. In this economy the party system is centralized, the leaders of nationwide parties face multiple electoral contests and nominate a list of candidates who propose policies that maximize the party's share of the vote in elections in districts i and $-i$. In a centralized party system, the dominant strategy of party leaders is to nominate to the closed list only those candidates who will carry out the party's ideal policies for districts i and $-i$ in the legislative game of local legislatures.

In the first stage of the local election of district i parties $z = \{L, M, R\}$ announce policy platforms for districts i and $-i$ and a list of candidates $\Theta_{CL}^{zi} = \left\{N_j^{zi}\right\}_{j=1}^{J}$ $\forall i, -i$ where N_l^{zi} is a

[6] To see this, note that the equalization of the net marginal benefits from local public goods in districts i and $-i$ is characterized by the expression

$$\int_{\forall e^i} h^i(e^i)\frac{\partial \mu^i}{\partial G^i}de^i + k^i\int_{\forall e^{-i}} h^{-i}(e^{-i})\frac{\partial \mu^{-i}}{\partial G^{-i}}de^{-i} - \left\{\frac{1}{N}\right\}\int_{\forall e^i} h^i(e^i)\frac{\partial \mu^i}{\partial x^i}de^i =$$
$$-\int_{\forall e^{-i}} h^{-i}(e^{-i})\frac{\partial \mu^{-i}}{\partial G^{-i}}de^{-i} - k^{-i}\int_{\forall e^i} h^i(e^i)\frac{\partial \mu^i}{\partial G^i}de^i + \left\{\frac{1}{N}\right\}\int_{\forall e^{-i}} h^{-i}(e^{-i})\frac{\partial \mu^{-i}}{\partial x^{-i}}de^{-i}$$

Re-arrange terms to show this is condition (8)

$$\sum_{\forall i,-i}(1 + k^{-i})\int_{\forall e^i} h^i(e^i)\frac{\partial \mu^i}{\partial G^i}de^i = \left\{\frac{1}{N}\right\}\sum_{\forall i,-i}\int_{\forall e^i} h^i(e^i)\frac{\partial \mu^i}{\partial x^i}de^i$$

sequence reflecting the name of candidate of party z in district i with order $j = 1,2 \ldots J$. If a party receives three seats in the local legislature the names of candidates $N_1^{zi}, N_2^{zi}, N_3^{zi}$ occupy those three seats.

During the second stage of the local electoral process, individuals vote for the party that advances the spending policy that is closest to their own views on public spending. In the third stage of the game, the local election takes place and parties receive a proportion of the seats in the local legislature equivalent to the proportion of votes received in the local election. In the fourth stage, the legislative bargaining takes place. We assume an exogenously given policy $\mathbf{g_{LC0}} = [g_{LC0}^{i}, g_{LC0}^{-i}]$ as the status quo. As before, we consider the following legislative procedure: the party with the highest share of the vote in the local election proposes a policy. If this party has a majority of the seats this policy is implemented. If no party has a majority of the seats in the local legislature then one party is selected randomly and proposes a policy. If this policy receives a majority of the votes then the policy is implemented. If this policy does not receive the majority then the status quo is implemented.

The expected proportion of the vote of party z in district i in the local election is $\phi_{cL}^{zi} = \int_{\forall\, e^i} h^i(e^i)\, F_c^{zi}(\Psi_{cL}^{zMi}, \Psi_{cL}^{zRi}) de^i$ where $\Psi_{cL}^{zMi} = v^{zi}(e^i, g_{cL}^{zi}) - v^{Mi}(e^i, g_{cL}^{Mi})$ where Ψ_{cL}^{zMi} is the difference in the voter's payoff with endowment e^i in district i if party z is elected and implements policy g_{cL}^{zi} in district i instead of the alternative policy g_{cL}^{Mi} when party M is elected. Similarly consider $\Psi_{cL}^{zRi} = v^{zi}(e^i, g_{cL}^{zi}) - v^{Ri}(e^i, g_{cL}^{Ri})$. We assume ϕ_{cL}^{zi} is continuous and strictly concave with respect g_{cL}^{zi}. The expected proportion of the vote for party z in both districts is $\phi_c^z = \sum_{\forall i,-i} \phi_{cL}^{zi}$.

The sub-game perfect Nash equilibrium of this electoral-legislative game is characterized by proposition 3.

Proposition 3. *The sub-game perfect Nash equilibrium for the fiscally decentralized provision of local public goods in economies with PR electoral systems and closed party lists is given by:*

3.i) In the first stage of local elections in district i and $-i$ party z proposes $\mathbf{g}_{\mathbf{cL}}^{**} = \left[g_{cL}^{**zi}, g_{cL}^{**z,-i}\right]$ and a list of candidates $\boldsymbol{\Theta}_{CL}^{z} = \left[\Theta_{CL}^{zi}, \Theta_{CL}^{z,-i}\right]$ who agree with this policy

$$\mathbf{g}_{\mathbf{cL}}^{**} \in argmax \ \ \phi_c^z = \sum_{\forall i,-i} \phi_{cL}^{zi} \tag{9}$$

3.ii) In the second stage of the game, a voter type e^i in district i votes for party z if

$$\begin{aligned} \Psi_{cL}^{zMi} &= v^{zi}\left(e^i, g_{cL}^{***zi}\right) - v^{Mi}\left(e^i, g_{cL}^{***Mi}\right) > 0 \\ \Psi_{cL}^{zRi} &= v^{zi}\left(e^i, g_{cL}^{***zi}\right) - v^{Ri}\left(e^i, g_{cL}^{***Ri}\right) > 0 \end{aligned} \tag{10}$$

Where g_{cL}^{***zi} is the policy to be implemented in the local legislative process.[7] Otherwise the voter votes for party M or party R.

3.iii) In the third stage of the game votes are counted and the shares of the votes and the seats are determined.

3.iv) In the fourth stage the status quo policy is given by g_0^i in district i and the legislative bargaining game takes place as follows:

a. If $\phi_{cL}^{zi} > \frac{1}{2}$ for some party z then $g_{cL}^{***zi} = g_{cL}^{**zi}$ is proposed and approved by majority.

b. If $\phi_{cL}^{zi} < \frac{1}{2}$ $\forall$ $z = \{L, M, R\}$ then randomly a party $z = \{L, M, R\}$ in district i proposes:

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[7] Note that the policy platform is given by g_{cL}^{**zi} while the policy proposed at the legislative game is given by g_{cL}^{***zi}.

- *A policy* $g_{cL}^{***zi} = g_0^i$ *if* $\left|g_0^i - g_{cL}^{***zi}\right| < \left|g_{MV}^i - g_{cL}^{***zi}\right| \quad \forall i$ *where* g_{MV}^i *is the median voter policy in district i.*
- *Otherwise party z proposes a policy* $g_{cL}^{***zi} = g_{MV}^i$ *if* $\left|g_0^i - g_{cL}^{***zi}\right| \geq \left|g_{MV}^i - g_{cL}^{***zi}\right|$.

Lemma 2 *In a system of local governments with a proportional representation electoral system and closed party lists, parties choose a policy platform with a set of Pareto efficient local public goods with and without spillovers* $\mathbf{g}_{\mathbf{cL}}^{**} = \left[g_{cL}^{**i}, g_{cL}^{**-i}\right]$, *where* $g_{cL}^{**i}\ \forall i, \forall z$ *satisfies the following:*

$$\int\limits_{\forall\, e^i} h^i(e^i)\frac{\partial \mu^i}{\partial G^i}de^i + k^{-i}\int\limits_{\forall\, e^{-i}} h^{-i}(e^{-i})\frac{\partial \mu^{-i}}{\partial G^{-i}}de^{-i} = \left\{\frac{1}{N}\right\}\int\limits_{\forall\, e^i} h^i(e^i)\frac{\partial \mu^i}{\partial x^i}de^i \qquad (11)$$

Proof

The problem for party z is to design a policy such that:

$$\underset{\{g_{cL}^{zi}, g_{cL}^{z,-i}\}}{Max}\left\{\int\limits_{\forall\, e^i} h^i(e^i)\, F_{cL}^{zi}\left(\Psi_{cL}^{zMi}, \Psi_{cL}^{zRi}\right)de^i + \int\limits_{\forall\, e^{-i}} h^{-i}(e^{-i})\, F_{cL}^{z,-i}\left(\Psi_{cL}^{zM,-i}, \Psi_{cL}^{zR,-i}\right)de^{-i}\right\} \quad (12)$$

The first order conditions imply

$$\int\limits_{\forall\, e^i} h^i(e^i)\left\{\frac{\partial F_c^{zi}}{\partial \Psi_c^{zMi}}\frac{\partial \Psi_c^{zMi}}{\partial g_{cL}^{zi}} + \frac{\partial F_c^{zi}}{\partial \Psi_c^{zRi}}\frac{\partial \Psi_c^{zRi}}{\partial g_{cL}^{zi}}\right\}de^i$$

$$+\int\limits_{\forall\, e^{-i}} h^{-i}(e^{-i})\left\{\frac{\partial F_c^{z,-i}}{\partial \Psi_c^{zM,-i}}\frac{\partial \Psi_c^{zM,-i}}{\partial g_{cL}^{zi}} + \frac{\partial F_c^{z,-i}}{\partial \Psi_c^{zR,-i}}\frac{\partial \Psi_c^{zR,-i}}{\partial g_{cL}^{zi}}\right\}de^i = 0 \quad \forall g_{cL}^{**zi} > 0, \forall i, \forall z \quad (13)$$

It is simple to show that $\frac{\partial \Psi_c^{zMi}}{\partial g_{cL}^{zi}} = \frac{\partial \Psi_c^{zRi}}{\partial g_{cL}^{zi}} = \frac{\partial \upsilon^{zi}}{\partial g_{cL}^{zi}}$ and $\frac{\partial \Psi_c^{zM,-i}}{\partial g_{cL}^{zi}} = \frac{\partial \Psi_c^{zR,-i}}{\partial g_{cL}^{zi}} = \frac{\partial \upsilon^{z,-i}}{\partial g_{cL}^{zi}}$. Moreover, the convergence of the parties' policies implies $\frac{\partial F_c^{zi}}{\partial \Psi_c^{zMi}} = \frac{\partial F_c^{zi}}{\partial \Psi_c^{zRi}} = \frac{\partial F_c^{z,-i}}{\partial \Psi_c^{zM,-i}} = \frac{\partial F_c^{z,-i}}{\partial \Psi_c^{zR,-i}} = f_c^{zi}(0) = f_c^{z,-i}(0) \in \mathbb{R}_+ \ \forall\, i, \forall\, z$. Therefore equation (13) becomes

$$\int_{\forall\, e^i} h^i(e^i)\frac{\partial \upsilon^{zi}}{\partial g_{cL}^{zi}} de^i + \int_{\forall\, e^{-i}} h^{-i}(e^{-i})\frac{\partial \upsilon^{z,-i}}{\partial g_{cL}^{zi}} de^i = 0 \quad \forall g_{cL}^{**zi} > 0, \forall i, \forall z \tag{14}$$

Use $\frac{\partial \upsilon^{zi}}{\partial g_c^{zi}} = \frac{\partial \mu^i}{\partial g_{cL}^{zi}} - \left(\frac{1}{N}\right)\frac{\partial \mu^i}{\partial x^i}$ and $\frac{\partial \upsilon^{z,-i}}{\partial g_c^{zi}} = k^{-i}\frac{\partial \mu^{-i}}{\partial g_{cL}^{zi}}$ to show that $g_{cL}^{**zi} > 0, \forall i, \forall z$ satisfies

$$\int_{\forall\, e^i} h^i(e^i)\frac{\partial \mu^i}{\partial g_{cL}^{zi}} de^i + k^{-i}\int_{\forall\, e^{-i}} h^{-i}(e^{-i})\frac{\partial \mu^{-i}}{\partial g_{cL}^{zi}} de^{-i} = \left\{\frac{1}{N}\right\}\int_{\forall\, e^i} h^i(e^i)\frac{\partial \mu^i}{\partial x^i} de^i \tag{15}$$

■

Lemma 2 says that parties in proportional representation systems with closed party lists, parties have incentives to design policy to maximize the parties' proportion of the expected voted in elections i and $-i$. This means that even under the case of local elections, parties have electoral incentives to internalize the benefits of local public goods provided by district i on other districts since the marginal probability of the vote for party z of a resident of district $-i$ depends on spillovers of local public goods. The expression in (11) says that parties select local public spending in district i where the marginal benefits of g_{cL}^{**zi} for residents of distric i $\int_{\forall\, e^i} h^i(e^i)\frac{\partial \mu^i}{\partial g_{cL}^{zi}} de^i$ plus the externality of local public goods on residents of district $-i$ $k^{-i}\int_{\forall\, e^{-i}} h^{-i}(e^{-i})\frac{\partial \mu^{-i}}{\partial g_{cL}^{zi}} de^{-i}$ are equal to the marginal costs of providing g_{cL}^{**zi}, that is, $\left\{\frac{1}{N}\right\}\int_{\forall\, e^i} h^i(e^i)\frac{\partial \mu^i}{\partial x^i} de^i$.

Proposition 4. *In local elections in districts* i *and* $-i$, *parties* $z = \{L, M, R\}$ *converge in selecting* g_c^{**zi} $\forall z$ *for an economy with a PR system and closed party lists. Therefore at the legislative stage of the local government in district* i, *the policy* $g_{cL}^{***zi} = g_{cL}^{**zi}\ \forall z, \forall i$ *is implemented by unanimity.*

Proof

The function of the share of the vote ϕ_{cL}^{z} is continuous and strictly concave. Hence $\mathbf{g}_{\mathbf{cL}}^{**} \in argmax\ \phi_{cL}^{z} = \sum_{\forall i,-i} \int_{\forall\, e^i} h^i(e^i)\, F_{cL}^{zi}(\Psi_{cL}^{zMi}, \Psi_{cL}^{zRi}) de^i$ is unique which implies parties converge in selecting policy platforms $g_{cL}^{**Li} = g_{cL}^{**Mi} = g_{cL}^{**Ri}\ \forall i$**. C**onvergence also implies that a random party $z = \{L, M, R\}$ in district i proposes g_c^{**zi} which is approved by unanimity. Hence, at the fourth stage of the game $g_{cL}^{***zi} = g_{cL}^{**zi}\ \forall z, \forall i$.

Theorem 1 *The Strong and Conventional Decentralization Theorems hold in PR Systems with Closed Party Lists***.** *That is, the provision of local public goods with and without inter-regional spillovers by a system of local governments in economies with PR electoral systems and closed party lists welfare-dominates the fiscally centralized provision.*

Proof

We first obtain the Pareto efficient local public goods. For that purpose consider a nationwide social welfare function given by[8]

[8] Recall that the preferences of an individual with an endowment e^i in district i are given by $v^i(e^i, G^i) = Max\ \mu^i(x^i, g^i, g^{-i}) = x^i G^i$ subject to $a)\ x^i = e^i - t^i$ and $b)\ g^i = Nt^i\ \forall i$, where $v^i(e^i, G^i)$ is the individual´s indirect utility, $\mu^i(x^i, g^i, g^{-i})$ are his preferences over a private good x^i, $G^i = (g^i + k^{-i} g^{-i})$ is the overall consumption of local public goods provided by district i, g^i, and by district $-i$, g^{-i}, and t^i is a head tax on residents of district i.

$$\Psi = \sum_{\forall\, i,-i} \int_{\forall\, e^i} h^i(e^i)\, v^i(e^i, G^i) de^i \tag{16}$$

Assume Ψ is strictly concave with respect g^i, g^{-i}. Let $\hat{\mathbf{g}}^* = [\hat{g}^{*i}, \hat{g}^{*,-i}]$ and $\hat{\mathbf{g}}^* \in argmax\,\Psi$. The first order condition implies $\hat{g}^{*i} > 0\ \forall i$ satisfies[9]

$$\int_{\forall\, e^i} h^i(e^i) \frac{\partial \mu^i}{\partial G^i} de^i + k^{-i} \int_{\forall\, e^{-i}} h^{-i}(e^{-i}) \frac{\partial \mu^{-i}}{\partial G^{-i}} de^{-i} = \left\{\frac{1}{N}\right\} \int_{\forall\, e^i} h^i(e^i) \frac{\partial \mu^i}{\partial x^i} de^i \tag{17}$$

Recall $\mathbf{g}_{\mathbf{c}}^{**\mathbf{z}}$ and $\mathbf{g}_{\mathbf{cL}}^{**\mathbf{z}}$ are, respectively the fiscally centralized and decentralized provisions of local public goods with and without spillovers. Condition (4) of Lemma 1 and condition (11) of Lemma 2 means that $\mathbf{g}_{\mathbf{cL}}^{**\mathbf{z}} = \hat{\mathbf{g}}^*$ and $\mathbf{g}_{\mathbf{c}}^{**\mathbf{z}} \neq \hat{\mathbf{g}}^*$ which implies

$$\Psi(\mathbf{g}_{\mathbf{cL}}^{**\mathbf{z}}) > \Psi(\mathbf{g}_{\mathbf{c}}^{**\mathbf{z}}) \tag{18}$$

The main result of Theorem 1 is that the decentralized provision of local public goods with and without externalities is welfare superior to the centralized provision for an economy with a PR electoral system and closed party lists. Theorem 1 shows that a system of local governments welfare-dominates the centralized provision when local public goods do not show spillovers leading to the satisfaction of the conventional Decentralization Theorem (see Oates 1972, 1995). Theorem 1 also shows that a system of local governments welfare-dominates the centralized provision even if local public goods show inter-regional spillovers. We call this result the Strong Decentralization Theorem.

To see this, note that if parties participate in elections of districts i and $-i$ then these parties have incentives to maximize their share of the vote in multiple electoral contests leading

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[9] For the case of $\hat{g}^{*-i} > 0\ \forall i$ this condition is

$$\int_{\forall\, -} h^{-i}(e^{-i}) \frac{\partial \mu^{-i}}{\partial G^{-i}} de^{-i} + k^i \int_{\forall\, e^i} h^i(e^i) \frac{\partial \mu^i}{\partial G^{-i}} de^i = \left\{\frac{1}{N}\right\} \int_{\forall\, e^{-i}} h^{-i}(e^{-i}) \frac{\partial \mu^{-i}}{\partial x^{-i}} de^{-i} \tag{17'}$$

parties to seek to maximize their nationwide share of votes. For this reason, parties have incentives to internalize the benefits of local public goods with inter-regional spillovers. This is the case because a party z recognizes that a change in the provision of the local public good of district i affects the probability that a voter in district $-i$ votes for the party in the election of that district, therefore party z has electoral incentives to incorporate the spillover effect of local public goods provided by district i in other districts. This means that the fiscally decentralized provision of local public goods with and without inter-regional externalities is Pareto efficient in a PR electoral system with closed party lists.

Since parties can nominate candidates to the list then this political system is centralized which allows parties to create incentives for candidates to adopt the party's policies instead of the ideal policy of a candidate running in district i that seeks to target local public spending to the interest of the average voter in each district. This last policy is not Pareto efficient if local public goods exhibit inter-regional spillovers.

Theorem 1 also says that a fiscally decentralized provision of local public goods can accommodate the heterogeneous preferences of individuals for local public goods since the decentralized provision of local public goods can differentiate local public spending according to the inter-regional heterogeneity of tastes while the centralized provision would be uniform.

IV Proportional Representation with Open Party List Systems and Fiscal Centralization

In this section we incorporate the analysis of the provision of local public goods (LPG's) in proportional representation (PR) systems with open party lists. The only distinction of the model of this section with the model in section I is that parties propose a list of candidates to hold a seat in the nationwide legislature but voters determine the order of the candidate in the list. As in section I, under a central government, local public goods are provided by a single government that represents voters of all districts. The government finances its expenditures through a uniform tax on residents of all districts. Local public goods provided by the central government are uniform across districts.

The electoral-legislative game has the following structure: in the first stage parties announce a policy platform and a list of candidates characterized by $\Theta^{z}_{open} = \left[\Theta^{zi}, \Theta^{z,-i}\right]$ where $\Theta^{zi} \in \mathbb{R}^{J}\colon \Theta^{zi} = \left[N^{zi}_{1} \dots N^{zi}_{J}\right]$ is a sequence N^{zi}_{j} showing the name of the candidate of party z in district i. The list does not imply any order (ranking) of candidates. The party also announces a policy platform that seeks to maximize the party's nationwide share of the vote in the election. In this stage, $J' \geq J$ candidates of all parties also announce their policy platforms.

In the second stage, individuals observe the parties' and the candidates' platforms. Voters vote for a candidate of some party that advances the spending policy that is closest to their own views on public spending. In the third stage, the election takes place and parties receive a proportion of the seats in the legislature equivalent to the proportion of votes received in the election. The relative position of candidates in the list of some party z is determined by the number of votes that each candidate of that party receives. The candidate with the largest number of votes gets the first position in the list, the candidates with the second largest number of votes obtains the second position in the list, and so on.[10]

[10] An equivalent way to rank candidates when all individuals vote is that the candidate with the largest proportion of votes (that is the ratio between the number of votes the candidate receives and the total number of votes for all candidates) gets the first position in the list, the candidates with the second largest proportion of votes obtains the second position in the list, and so on.

In the fourth stage, the legislative bargaining takes place. We assume an exogenously given policy $\mathbf{g_0} = [g_0^i, g_0^{-i}]$ as the status quo. We consider the next legislative procedure: If one party has a majority of the seats then one member of this party proposes a policy. If this policy receives at least a majority of the votes then the policy is implemented. If no party has the majority of the seats, then one member of the legislature is selected randomly and proposes a policy. If this policy receives a majority of the votes then the policy is implemented. If this policy does not receive the majority then the status quo is implemented.

For economies with open party list systems there is less political centralization relative the case of closed party list systems. This is the case, because candidates not only need to please party leaders but also need to get votes from the electorate to secure a high position in the party's list so they can get a seat in the legislature. Parties adopt a policy platform on public spending that seeks to maximize the proportion of votes that these parties can get in the nationwide election, while candidates announce a binding policy platform that maximizes the joint probability of obtaining the nomination of the party and the highest number of the votes from the electorate to secure a high position in the list that might lead the candidate to hold a seat in the legislature.[11]

The joint probability of candidate j in party z in district i of obtaining the party's nomination and the highest proportion of votes from the electorate is given by $Pr^{jzi} =$

[11] For simplicity of the analysis, in this paper we ignore dynamic inconsistency issues that might arise because of conflicts of objectives of candidates at the different stages of the electoral-legislative game. Dynamic inconsistency issues might induce candidates to shift policy platforms between the first and second stage. However, these shifting strategies from candidates might entail electoral costs since voters might adopt strategies, such as the abstention of the vote for those candidates who cannot commit to a policy, to try to preclude candidates to adopt non-binding policy platforms. To avoid this electoral cost, candidates might find attractive to adopt a binding policy platform such as announcing a policy that maximizes the joint probability of obtaining the nomination of the party and the highest number of the votes from the electorate.

$Pr^{jzi}\left(d^i,\{\Psi_c^{jzi-1Mi}(e^i) \ldots.. \ \Psi_c^{jzi-JMi}(e^i)\}_{\forall e^i},\{\Psi_c^{jzi-1Ri}(e^i) \ldots.. \ \Psi_c^{jzi-JRi}(e^i)\}_{\forall e^i}\right)$ where $d^i = \left|g_{co}^{jzi} - g_{co}^{**zi}\right|$ is the distance between the ideal policy of party z in district i, g_c^{**zi} (the policy that maximizes the party's share of the vote in the election), and the policy adopted by candidate g_{co}^{jzi}. The function Pr^{jzi} is strictly decreasing with d^i (the higher d^i the less likely candidate j will get the nomination by party z).

Candidate j of party z is not only competing for the nomination but also is competing with candidates $j = 1, \ldots J$ of parties M and R for votes from the electorate.[12] The joint probability Pr^{jzi} also depends on a sequence of the welfare calculus of voters determined by the sequences $\{\Psi_c^{jzi-1Mi}(e^i) \ldots.. \ \Psi_c^{jzi-JMi}(e^i)\}_{\forall e^i}$ and $\{\Psi_c^{jzi-1Ri}(e^i) \ldots.. \ \Psi_c^{jzi-JRi}(e^i)\}_{\forall e^i}$ with $\Psi_c^{jzi-1Mi}(e^i) = v^{jzi}(e^i, g_{co}^{jzi}) - v^{1Mi}(e^i, g_{co}^{1Mi})$ where $\Psi_c^{jzi-1Mi}$ is the difference in the payoff of voter type e^i in district i if candidate j of party z is elected and implements policy g_{co}^{jzi} in district i instead of the alternative policy g_{co}^{1Mi} when candidate 1 of party M is elected. A similar interpretation is given to $\Psi_c^{jzi-2Mi} \ldots \Psi_c^{jzi-JMi}$ and $\Psi_c^{jzi-1Ri} \ldots \ \Psi_c^{jzi-JRi}$. Voter type e^i votes for candidate j of party z if $\Psi_c^{jzi-1Mi}(e^i) > 0 \ldots.. \ \Psi_c^{jzi-JMi}(e^i) > 0$ and $\Psi_c^{jzi-1Ri}(e^i) > 0 \ldots.. \ \Psi_c^{jzi-JRi}(e^i) > 0$. Voting is sincere and sequentially rational. We assume that the function Pr^{jzi} is continuous and strictly concave.

As we mentioned before, party z announces a policy platform that seeks to maximize the party's share of the vote in the election ϕ_c^z where $\phi_c^z = \sum_{\forall i,-i} \int_{\forall e^i} h^i(e^i)\, F_c^{zi}(\Psi_c^{jzi-1Mi} \ldots \ \Psi_c^{jzi-JMi}, \Psi_c^{jzi-1Ri} \ldots \ \Psi_c^{jzi-JRi})\, de^i$ and

[12] For simplicity we assume there are J candidates in each party. However, our results hold if we assume that each party has a different number of candidates.

$F_c^{zi}\left(\Psi_c^{jzi-1Mi} \ldots\ \Psi_c^{jzi-JMi}, \Psi_c^{jzi-1Ri} \ldots\ \Psi_c^{jzi-JRi}\right)$ is the probability that a voter type e^i in district i votes for party z in the nationwide election.

Formally the sub-game perfect Nash equilibrium of this electoral-legislative game is characterized by proposition 5.

Proposition 5 *The sub-game perfect Nash equilibrium for the fiscally centralized provision of*

5.i) In the first stage party z announces a policy platform $\mathbf{g_{co}^{**z}} = \left[g_{co}^{**zi}, g_{co}^{**z,-i}\right]$ *and a list of candidates* $\Theta_{open}^{*z} = \left[\Theta^{*jzi}, \Theta^{*jz,-i}\right]$ *where* $\Theta^{*jzi} = \left\{N_j^{zi}\right\}_{j=1}^{J}$ *is a sequence* N_j^{zi} *reflecting the name of candidate* j *of party* z *in district* i *who agrees with the party's policy such that*[13]

$$\mathbf{g_{co}^{**z}} \in argmax\ \ \phi_c^z$$

Where

$$\phi_c^z = \sum_{\forall i,-i} \int_{\forall\, e^i} h^i\left(e^i\right) F_c^{zi}\left(\Psi_c^{jzi-1Mi} \ldots\ \Psi_c^{jzi-JMi}, \Psi_c^{jzi-1Ri} \ldots\ \Psi_c^{jzi-JRi}\right) de^i$$

$$subject\ \ to\ \ g_{co}^{zi} = g_{co}^{z,-i}\ \ \forall\, z \tag{19}$$

5.i.2) Candidates $j = 1, ..J$ *of party* $z = \{L, M, R\}$ *propose a policy platform* g_{co}^{**jzi} *such that*

$$g_{co}^{**jzi} \in argmax\ \ Pr^{jzi}$$

Where

[13] The notation $\mathbf{g_{co}^{**z}}$ is to distinguish the ideal policy platform of a party z in a fiscally centralized economy with a PR system and *open party lists* from the ideal policy platform of a party z in a fiscally centralized economy with a PR system and *closed party lists* $\mathbf{g_c^{**z}}$.

$$Pr^{jzi} = Pr^{jzi}\left(d^i, \left\{\Psi_c^{jzi-1Mi}\left(e^i\right) \dots\ \Psi_c^{jzi-JMi}\left(e^i\right)\right\}_{\forall e^i} \left\{\Psi_c^{jzi-1Ri}\left(e^i\right) \dots\ \Psi_c^{jzi-JRi}\left(e^i\right)\right\}_{\forall e^i}\right)$$

$$subject\ \ to\colon\ g_{co}^{jzi} = g_{co}^{jz,-i} = g_{co}^{jz} \quad \forall j, \forall z, \forall i$$

$$and \quad d^i = \left|g_{co}^{jzi} - g_{co}^{**zi}\right| \quad \forall i \tag{20}$$

5.ii) In the second stage of the game, a voter type e^i in district i votes for candidate j of party z if

$$\begin{gathered}
\Psi_c^{jzi-1Mi}\left(g_{co}^{***jzi}, g_{co}^{***1Mi}\right) = \upsilon^{jzi}\left(e^i, g_{co}^{jzi}\right) - \upsilon^{1Mi}\left(e^i, g_{co}^{1Mi}\right) > 0 \\
\vdots \\
\Psi_c^{jzi-JMi}\left(g_{co}^{***jzi}, g_{co}^{***JMi}\right) = \upsilon^{jzi}\left(e^i, g_{co}^{jzi}\right) - \upsilon^{1Mi}\left(e^i, g_{co}^{JMi}\right) > 0 \\
\\
\vdots \\
\\
\Psi_c^{jzi-1Ri}\left(g_{co}^{***jzi}, g_{co}^{***1Ri}\right) = \upsilon^{jzi}\left(e^i, g_{co}^{jzi}\right) - \upsilon^{1Mi}\left(e^i, g_{co}^{1Ri}\right) > 0 \\
\vdots \\
\Psi_c^{jzi-JRi}\left(g_{co}^{***jzi}, g_{co}^{***JRi}\right) = \upsilon^{jzi}\left(e^i, g_{co}^{jzi}\right) - \upsilon^{1Mi}\left(e^i, g_{co}^{JRi}\right) > 0
\end{gathered} \tag{21}$$

Otherwise the voter votes for another candidate of party M or party R.

5.iii) In the third stage, votes are counted, the order of candidates $j = 1,2,,,J \in z = \{L, M, R\}$ in each party's list is determined by the number of votes each candidate j of party z receives, and the shares of the votes and seats are determined.

5.iv) In the fourth stage the status quo policy is given by $\mathbf{g}_\mathbf{0}^* = \left[g_0^i, g_0^{-i}\right]$ and the legislative bargaining game takes place as follows:

a. If $\phi_c^z > \frac{1}{2}$ for some party z then a random candidate j of party z proposes $g_{co}^{***jzi} = g_{co}^{**zi}\ \forall i, -i$ and this policy is implemented by majority.

b. If $\phi_c^z < \frac{1}{2}\ \forall\ z = \{L, M, R\}$ then randomly a candidate j of party z in district i proposes:

- *A policy* $g_{co}^{***jzi} = g_0^i \ \ if \ \ \left|g_0^i - g_{co}^{***jzi}\right| < \left|g_{MV}^i - g_{co}^{***jzi}\right| \ \forall i$ *where* g_{MV}^i *is the median voter policy in district* i.
- *Otherwise candidate* j *of party* z *proposes a policy* $g_{co}^{***zi} = g_{MV}^i \ \ if$

$$\left|g_0^i - g_c^{***jzi}\right| \geq \left|g_{MV}^i - g_c^{***jzi}\right|$$

Recall that the joint probability of obtaining the nomination of the party and a seat in the legislature for candidate j of party z in district i, Pr^{jzi}, is continuous and strictly concave. Hence $g_{co}^{**jzi} \in argmax\, Pr^{jzi}\left(d^i, \left\{\Psi_c^{jzi-1Mi} \ldots\ldots \ \Psi_c^{jzi-JMi}\right\}_{\forall e^i}, \left\{\Psi_c^{jzi-1Ri} \ldots\ldots \ \Psi_c^{jzi-JRi}\right\}_{\forall e^i}\right)$ subject to $g_{co}^{zi} = g_{co}^{z,-i} = g_{co}^{jz} \ \forall j, \forall z, \forall i, -i$ is unique which implies that all candidates of all parties converge in selecting a policy platform, that is, $g_{co}^{**jLi} = g_{co}^{**jMi} = g_{co}^{**jRi} \ \forall j = 1, \ldots J, \forall i, -i$.

Lemma 3 *Local public goods with and without spillovers for an economy with a PR electoral system with open party lists and a fiscally centralized government are provided as follows: Define* $\eta^{jzi} \in [0,1]$ *as the ratio between the marginal effect of changes in* d^i *in the joint probability of candidate* j *of party* z *in district* i *to obtain the nomination and a seat in the national legislature* $\frac{\partial Pr^{jzi}}{\partial d^i}$ *and the sum of* $\frac{\partial Pr^{jzi}}{\partial d^i}$ *and* $\frac{\partial Pr^{jzi}}{\partial \Psi_c^{jzi-1Mi}} + \cdots \frac{\partial Pr^{jzi}}{\partial \Psi_c^{jzi-JMi}} + \frac{\partial Pr^{jzi}}{\partial \Psi_c^{jzi-1Ri}} + \cdots \frac{\partial Pr^{jzi}}{\partial \Psi_c^{jzi-JRi}}$ *where* $\frac{\partial Pr^{jzi}}{\partial \Psi_c^{jzi-1Mi}} + \cdots \frac{\partial Pr^{jzi}}{\partial \Psi_c^{jzi-JMi}} + \frac{\partial Pr^{jzi}}{\partial \Psi_c^{jzi-1Ri}} + \cdots \frac{\partial Pr^{jzi}}{\partial \Psi_c^{jzi-JRi}}$ *is the marginal effect of candidate j of party z of obtaining a sufficient amount of votes to secure a seat in the legislature*

$$\eta^{jzi} = \frac{\dfrac{\partial Pr^{jzi}}{\partial d^i}}{\dfrac{\partial Pr^{jzi}}{\partial d^i} + \dfrac{\partial Pr^{jzi}}{\partial \Psi_c^{jzi-1Mi}} + \cdots \dfrac{\partial Pr^{jzi}}{\partial \Psi_c^{jzi-JMi}} + \dfrac{\partial Pr^{jzi}}{\partial \Psi_c^{jzi-1Ri}} + \cdots \dfrac{\partial Pr^{jzi}}{\partial \Psi_c^{jzi-JRi}}} \tag{22}$$

$L3.i)$ $If\ \eta^{jzi} = 1$ *then* $g_c^{**jzi} = g_c^{**zi}$ *which implies that local public goods with and without spillovers are Pareto efficient and given by*

$$\sum_{\forall\, i,-i}\left(1+k^{-i}\right)\int_{\forall\, e^{i}} h^{i}\left(e^{i}\right)\frac{\partial\mu^{i}}{\partial G^{i}}de^{i}=\left\{\frac{1}{N}\right\}\sum_{\forall i,-\, i}\int_{\forall\, e^{i}} h^{i}\left(e^{i}\right)\frac{\partial\mu^{i}}{\partial x^{i}}de^{i}=0\quad \forall g_{c}^{**jzi}>0 \quad (23)$$

$L3.ii)$ If $\eta^{jzi}=0$ then local public goods are Pareto efficient only when local public goods do not show spillovers and are given by

$$\int_{\forall\, e^{i}} h^{i}\left(e^{i}\right)\frac{\partial\mu^{i}}{\partial G^{i}}de^{i}=\int_{\forall\, e^{i}} h^{i}\left(e^{i}\right)\frac{\partial\mu^{i}}{\partial x^{i}}de^{i}\qquad \forall g_{co}^{**jzi}>0 \qquad (24)$$

Proof

The problem of design of a policy platform for candidate j of party z in district i is

$$Max_{\left\{g_{co}^{jzi}\right\}}\, Pr^{jzi}\left(d^{i},\left\{\Psi_{c}^{jzi-1Mi}\left(e^{i}\right)\ldots\ \Psi_{c}^{jzi-JMi}\left(e^{i}\right)\right\}_{\forall e^{i}}\left\{\Psi_{c}^{jzi-1Ri}\left(e^{i}\right)\ldots\ \Psi_{c}^{jzi-JRi}\left(e^{i}\right)\right\}_{\forall e^{i}}\right) \ (25)$$

And we impose the constraint $g_{co}^{jzi}=g_{co}^{jz,-i}$ $\forall j,\forall z,\forall i,-i$ such that $\upsilon^{i}\left(e^{i},G^{i}\right)=Max\ \mu^{i}\left(x^{i},g^{i},g^{-i}\right)=x^{i}G^{i}=x^{i}\left(1+k^{-i}\right)g^{i}$ subject to $a)\ x^{i}=e^{i}-t^{i}$ and $b)\ g^{i}=Nt^{i}$ $\forall i.$

The first order conditions is:

$$\frac{\partial Pr^{jzi}}{\partial d^{i}}\frac{\partial d^{i}}{\partial g_{co}^{jzi}}+\sum_{\forall e^{i}}\left\{\frac{\partial Pr^{jzi}}{\partial\Psi_{c}^{jzi-1Mi}}\frac{\partial\Psi_{c}^{jzi-1Mi}\left(e^{i}\right)}{\partial g_{co}^{jzi}}+\cdots\ \frac{\partial Pr^{jzi}}{\partial\Psi_{c}^{jzi-JMi}}\frac{\partial\Psi_{c}^{jzi-JMi}\left(e^{i}\right)}{\partial g_{co}^{jzi}}\right\}$$

$$+\sum_{\forall e^{i}}\left\{\frac{\partial Pr^{jzi}}{\partial\Psi_{c}^{jzi-1Ri}}\frac{\partial\Psi_{c}^{jzi-1Ri}\left(e^{i}\right)}{\partial g_{co}^{jzi}}+\cdots\ \frac{\partial Pr^{jzi}}{\partial\Psi_{c}^{jzi-JRi}}\frac{\partial\Psi_{c}^{jzi-JRi}\left(e^{i}\right)}{\partial g_{co}^{jzi}}\right\}=0\quad \forall g_{co}^{**jzi}>0 \qquad (26)$$

It is simple to show that

$$\frac{\partial\Psi_{c}^{jzi-1Mi}\left(e^{i}\right)}{\partial g_{co}^{jzi}}\ldots=\frac{\partial\Psi_{c}^{jzi-JMi}\left(e^{i}\right)}{\partial g_{co}^{jzi}}=$$

$$\frac{\partial\Psi_{c}^{jzi-1Ri}\left(e^{i}\right)}{\partial g_{co}^{jzi}}\ldots\ \frac{\partial\Psi_{c}^{jzi-JRi}\left(e^{i}\right)}{\partial g_{co}^{jzi}}=\left(1+k^{-i}\right)\frac{\partial\mu^{i}\left(e^{i}\right)}{\partial g_{co}^{jzi}}-\left(1+k^{-i}\right)\frac{1}{N}\frac{\partial\mu^{i}\left(e^{i}\right)}{\partial x^{i}} \qquad (27)$$

And

$$\frac{\partial d^i}{\partial g_c^{jzi}} = \frac{g_{co}^{jzi} - g_{co}^{**zi}}{\left|g_{co}^{jzi} - g_{co}^{**zi}\right|} \quad \forall i \tag{28}$$

Define $\eta^{jzi} \in [0,1]$ where

$$\eta^{jzi} = \frac{\frac{\partial Pr^{jzi}}{\partial d^i}}{\frac{\partial Pr^{jzi}}{\partial d^i} + \frac{\partial Pr^{jzi}}{\partial \Psi_c^{jzi-1Mi}} + \cdots \frac{\partial Pr^{jzi}}{\partial \Psi_c^{jzi-JMi}} + \frac{\partial Pr^{jzi}}{\partial \Psi_c^{jzi-1Ri}} + \cdots \frac{\partial Pr^{jzi}}{\partial \Psi_c^{jzi-JRi}}} \tag{29}$$

Divide (26) by $\frac{\partial Pr^{jzi}}{\partial d^i} + \frac{\partial Pr^{jzi}}{\partial \Psi_c^{jzi-1Mi}} + \cdots \frac{\partial Pr^{jzi}}{\partial \Psi_c^{jzi-JMi}} + \frac{\partial Pr^{jzi}}{\partial \Psi_c^{jzi-1Ri}} + \cdots \frac{\partial Pr^{jzi}}{\partial \Psi_c^{jzi-JRi}}$ and re-express this condition as follows:

$$\eta^{jzi} \frac{g_{co}^{jzi} - g_{co}^{**zi}}{\left|g_{co}^{jzi} - g_{co}^{**zi}\right|} + \left\{1 - \eta^{jzi}\right\} \sum_{\forall e^i} \left\{\frac{\partial \mu^i(e^i)}{\partial g_{co}^{jzi}} - \frac{1}{N} \frac{\partial \mu^i(e^i)}{\partial x^i}\right\} = 0 \quad \forall g_c^{**jzi} > 0 \tag{30}$$

The convergence of the candidates' policies means that condition (26) can be expressed as follows:

$$\eta^{jzi} \frac{g_{co}^{jzi} - g_{co}^{**zi}}{\left|g_{co}^{jzi} - g_{co}^{**zi}\right|} + \left\{1 - \eta^{jzi}\right\}\left(1 + k^{-i}\right) \sum_{\forall e^i} \left\{\frac{\partial \mu^i(e^i)}{\partial g_{co}^{jzi}} - \frac{1}{N} \frac{\partial \mu^i(e^i)}{\partial x^i}\right\} = 0 \quad \forall g_{co}^{**jzi} > 0 \tag{31}$$

Clearly if $\eta^{jzi} = \eta^{jz,-i} = 1$ then $g_{co}^{jzi} = g_{co}^{**zi} = g_{co}^{**z,-i}$ and local public goods with and without spillovers are Pareto efficient and given by

$$\sum_{\forall i,-i} \left(1 + k^{-i}\right) \int_{\forall e^i} h^i(e^i) \frac{\partial \mu^i}{\partial G^i} de^i = \left\{\frac{1}{N}\right\} \sum_{\forall i,-i} \int_{\forall e^i} h^i(e^i) \frac{\partial \mu^i}{\partial x^i} de^i = 0 \quad \forall g_{co}^{**jzi} > 0 \tag{32}$$

If $\eta^{jzi} = \eta^{jz,-i} = 0$ then $\sum_{\forall e^i} \left\{\frac{\partial \mu^i(e^i)}{\partial g_c^{jzi}} - \frac{1}{N} \frac{\partial \mu^i(e^i)}{\partial x^i}\right\} = 0 \quad \forall g_{co}^{**jzi} > 0$ which implies

$$\int_{\forall e^i} h^i(e^i) \frac{\partial \mu^i}{\partial G^i} de^i = \int_{\forall e^i} h^i(e^i) \frac{\partial \mu^i}{\partial x^i} de^i \qquad \forall g_{co}^{**jzi} > 0 \tag{33}$$

In Lemma 3 the term $\eta^{jzi} \in [0,1]$ shows the relative importance for candidate j of party z in district i of obtaining the nomination versus obtaining the highest possible share of the vote from the electorate to guarantee a seat in the legislature. If $\eta^{jzi} = 1$, candidate j of party z in district i selects the ideal policy platform of party z $g_{co}^{**jzi} = g_{co}^{**zi}$ to try to secure the nomination because in this case the seat in the legislature is basically guaranteed. If $\eta^{jzi} = 0$ candidate j of party z in district i selects the ideal policy of the average voter of district i to secure the highest number of votes from the electorate.

However, for the case of $\eta^{jzi} = 0$, candidates of parties in each district do not recognize the external benefits on other districts of local public goods provided by district i. Therefore, local public goods are Pareto efficient only when local public goods do not have spillovers. For the general case $\eta^{jzi} \in (0,1)$, the candidates' policy platform reflects a convex combination of multiple objectives the candidate seeks to achieve. With a weight of $\eta^{jzi} > 0$ the candidate considers a policy that seeks to secure the party's nomination and with a weight $\{1 - \eta^{jzi}\} > 0$ the candidate considers the ideal policy of the average voter of district i on local public spending. This means that, in general (if $\eta^{jzi} > 0$), candidates will internalize to some extent inter-regional spillovers but this issue plays a minor (significant) role in policy design if the value of η^{jzi} is close to zero (one). Similarly, the weight the candidate assigns to the demands for local public spending of residents of district i plays a significant (minor) role in policy design if the value of η^{jzi} is close to zero (one).

V Proportional Representation Electoral Systems with Open Party Lists and Fiscal Decentralization

In this section, we extend our analysis of the provision of local pubic goods for an economy with a PR electoral system and open party lists when the economy is fiscally decentralized. In this case there are two local elections in districts i and $-i$. Policy is determined by the local legislature. In the first stage of the local election of district i parties $z = \{L, M, R\}$

announce policy platforms and a list of candidates $\Theta_{DL}^{zi} = \{N_j^{zi}\}_{j=1}^{J}$ $\forall i, -i$ where N_j^{zi} is a sequence reflecting the name of candidate of party z in district i. The list in this case does not reflect an order. In this stage, candidates also announce their policy platforms.

In the second stage, individuals observe the parties' and the candidates' platforms. In the third stage, the local election takes place and parties receive a proportion of the seats in the local legislature equivalent to the proportion of votes received in the election. The relative position of candidates in the list of some party z is determined by the number of votes that each candidate of that party receives. In the fourth stage, the local legislative bargaining takes place. We assume an exogenously given policy $\mathbf{g}_{\mathbf{0}}^{*} = [g_0^i, g_0^{-i}]$ in districts i and $-i$ as the status quo. We consider the next legislative procedure: If one party has a majority of the seats then one member of this party proposes a policy. If this policy receives at least a majority of the votes then the policy is implemented. If no party has the majority of the seats, then one member of the legislature of district i is selected randomly and proposes a policy. If this policy receives a majority of the votes then the policy is implemented. If this policy does not receive the majority then the status quo is implemented.

Proposition 7. *The sub-game perfect Nash equilibrium of the electoral-legislative game for economies with PR and open party lists systems and fiscal decentralization is characterized as follows:*

7.i) In the first stage of the local election in district i, party z announces a policy platform $\boldsymbol{g}_{\boldsymbol{DL}}^{**\boldsymbol{z}} = [g_{DL}^{**zi}, g_{DL}^{**z,-i}]$ and a list of candidates $\boldsymbol{\Theta}_{\boldsymbol{DL}}^{*\boldsymbol{z}} = [\Theta_{DL}^{zi}, \Theta_{DL}^{z,-i}]$ where ${\Theta_{DL}^{zi}}^{zi} = \{N_j^{jzi}\}_{j=1}^{J}$ is a *sequence* N_j^{jzi} *reflecting the name of candidate* j *of party* z *in district* i *who agrees with the party's policy such that*

$$\mathbf{g}_{\mathbf{DL}}^{**\mathbf{z}} \in argmax\ \phi_{DL}^{z}$$

Where

$$\phi_{DL}^{z} = \sum_{\forall i,-i} \int_{\forall\, e^i} h^i(e^i)\, F_c^{zi}\big(\Psi_c^{jzi-1Mi} \ldots\ \Psi_c^{jzi-JMi}, \Psi_c^{jzi-1Ri} \ldots\ \Psi_c^{jzi-JRi}\big) de^i \qquad (34)$$

7.i.2) Candidates $j = 1, .. J$ *of party* $z = \{L, M, R\}$ *in district* $i, \forall i$ *propose a policy platform* g_{DL}^{**jzi} *such that*

$$g_{DL}^{**jzi} \in argmax\ Pr_{DL}^{jzi}$$

Where

$$Pr_{DL}^{jzi} = Pr_{DL}^{jzi}\left(d^i, \{\Psi_c^{jzi-1Mi}(e^i) \ldots \Psi_c^{jzi-JMi}(e^i)\}_{\forall e^i} \{\Psi_c^{jzi-1Ri}(e^i) \ldots \Psi_c^{jzi-JRi}(e^i)\}_{\forall e^i}\right) \quad (35)$$

And $d^i = |g_{DL}^{jzi} - g_{DL}^{**zi}|$.

7.ii) In the second stage of the game, a voter type e^i *in district* i *votes for party* z *if*

$$\Psi_c^{jzi-1Mi}\big(g_{DL}^{***jzi}, g_{DL}^{***1Mi}\big) = v^{jzi}\big(e^i, g_{DL}^{***jzi}\big) - v^{1Mi}\big(e^i, g_{DL}^{***1Mi}\big) > 0$$
$$\vdots$$
$$\Psi_c^{jzi-JMi}\big(g_{DL}^{***jzi}, g_{DL}^{***JMi}\big) = v^{jzi}\big(e^i, g_{DL}^{***jzi}\big) - v^{1Mi}\big(e^i, g_{DL}^{***JMi}\big) > 0$$

$$\vdots$$

$$\Psi_c^{jzi-1Ri}\left(g_{DL}^{***jzi}, g_{DL}^{***1Ri}\right) = \upsilon^{jzi}\left(e^i, g_{DL}^{***jzi}\right) - \upsilon^{1Mi}\left(e^i, g_{DL}^{***1Ri}\right) > 0$$
$$\vdots \tag{36}$$
$$\Psi_c^{jzi-JRi}\left(g_{DL}^{***jzi}, g_{DL}^{***JRi}\right) = \upsilon^{jzi}\left(e^i, g_{DL}^{***jzi}\right) - \upsilon^{1Mi}\left(e^i, g_{DL}^{***JRi}\right) > 0$$

Otherwise the voter votes for another candidate of party M or party R.

7.iii) In the third stage, votes are counted and the shares of the votes and the seats of the local legislature are determined. The order of the list is determined by the number of votes each candidate j of party $z = \{L, M, R\}$ receives.

7.iv) In the fourth stage the status quo policy is given by $\mathbf{g_0} = \left[g_0^i, g_0^{-i}\right]$ and the legislative bargaining game of district i takes place as follows: a candidate j of some party z in district i is randomly chosen and proposes:

a. If $\phi_c^{zi} > \frac{1}{2}$ for some party z in district i then the candidate j proposes $g_{DL}^{***jzi} = g_{DL}^{**zi}$ $\forall i, -i$. This policy is approved by majority of the local legislature.

b. If $\phi_c^{zi} < \frac{1}{2}$ $\forall\, z = \{L, M, R\}$ then a random candidate j of party z in district i proposes:

- *A policy $g_{DL}^{***jzi} = g_0^i$ if $\left|g_0^i - g_{DL}^{***jzi}\right| < \left|g_{MV}^i - g_{DL}^{***jzi}\right|$ $\forall i$ where g_{MV}^i is the median voter policy in district i.*
- *Otherwise candidate j of party z proposes a policy $g_c^{***jzi} = g_{MV}^i$ if*

$$\left|g_0^i - g_{DL}^{***jzi}\right| \geq \left|g_{MV}^i - g_{DL}^{***jzi}\right|$$

Lemma 4 *In a PR electoral system with open party lists and fiscal decentralization, the ideal policy platform for party z is $\mathbf{g_{DL}^{**z}} = \left[g_{DL}^{**zi}, g_{DL}^{**z,-i}\right]$ such that*

$\mathbf{g_{DL}^{**z}} \in argmax\ \ \phi_{DL}^{z}$. *For the party's policy platform* $\mathbf{g_{DL}^{**z}}$ *local public goods with and without spillovers are Pareto efficient and satisfy the following:*

$$\int_{\forall\, e^i} h^i(e^i)\frac{\partial \mu^i}{\partial G^i}de^i + k^{-i}\int_{\forall\, e^{-i}} h^{-i}(e^{-i})\frac{\partial \mu^{-i}}{\partial G^{-i}}de^{-i}$$

$$= \left\{\frac{1}{N}\right\}\int_{\forall\, e^i} h^i(e^i)\frac{\partial \mu^i}{\partial x^i}de^i \qquad \forall g_{DL}^{**zi} > 0 \qquad (37)$$

Proof

The problem for party z is to design a policy such that:

$$Max_{\left\{g_{DL}^{**zi}\right\}}\quad \phi_{DL}^{z} = \int_{\forall\, e^i} h^i(e^i)\, F_c^{zi}\left(\Psi_c^{jzi-1Mi} \ldots\ \Psi_c^{jzi-JMi}, \Psi_c^{jzi-1Ri} \ldots\ \Psi_c^{jzi-JRi}\right)de^i$$

$$+\int_{\forall\, e^{-i}} h^{-i}(e^{-i})\, F_{cL}^{z,-i}\left(\Psi_c^{jz.-i-1M.-i} \ldots\ \Psi_c^{jz,-i-JM.-i}, \Psi_c^{jz,-i-1R,-i} \ldots\ \Psi_c^{jz,-i-JR,-i}\right)de^{-i} \qquad (38)$$

The first order condition for $\forall g_{DL}^{**zi} > 0$ implies

$$\int_{\forall\, e^i} h^i(e^i)\left\{\frac{\partial F_c^{zi}}{\partial \Psi_c^{zMi}}\frac{\partial \Psi_c^{jzi-1Mi}}{\partial g_{DL}^{**zi}} + \cdots \frac{\partial F_c^{zi}}{\partial \Psi_c^{zMi}}\frac{\partial \Psi_c^{jzi-JMi}}{\partial g_{DL}^{**zi}}\right\}de^i$$

$$+\int_{\forall\, e^i} h^i(e^i)\left\{\frac{\partial F_c^{zi}}{\partial \Psi_c^{jzi-1Ri}}\frac{\partial \Psi_c^{jzi-1Ri}}{\partial g_{DL}^{**zi}} + \cdots \frac{\partial F_c^{zi}}{\partial \Psi_c^{jzi-JRi}}\frac{\partial \Psi_c^{jzi-JRi}}{\partial g_{DL}^{**zi}}\right\}de^i$$

$$+\int_{\forall\, e^{-i}} h^{-i}(e^{-i})\left\{\frac{\partial F_c^{z,-i}}{\partial \Psi_c^{jz,-i-1M,-i}}\frac{\partial \Psi_c^{jz,-i-1M,-i}}{\partial g_{DL}^{**zi}} + \cdots \frac{\partial F_c^{z,-i}}{\partial \Psi_c^{jz,-i-JM,-i}}\frac{\partial \Psi_c^{jz,-i-JM,-i}}{\partial g_{DL}^{**zi}}\right\}de^i\ +$$

$$\int_{\forall\, e^{-i}} h^{-i}(e^{-i})\left\{\frac{\partial F_c^{z,-i}}{\partial \Psi_c^{jz,-i-1R,-i}}\frac{\partial \Psi_c^{jz,-i-1R,-i}}{\partial g_{DL}^{**zi}} + \cdots \frac{\partial F_c^{z,-i}}{\partial \Psi_c^{jz,-i-1R,-i}}\frac{\partial \Psi_c^{jz,-i-1R,-i}}{\partial g_{DL}^{**zi}}\right\}de^i = 0 \qquad (39)$$

Where

$$\frac{\partial \Psi_c^{jzi-1Mi}}{\partial g_{DL}^{**zi}} = \frac{\partial \Psi_c^{jzi-JMi}}{\partial g_{DL}^{**zi}} = \frac{\partial \Psi_c^{jzi-1Ri}}{\partial g_{DL}^{**zi}} = \frac{\partial \Psi_c^{jzi-JRi}}{\partial g_{DL}^{**zi}} = \frac{\partial \mu^i(e^i)}{\partial g_{DL}^{zi}} - \frac{1}{N}\frac{\partial \mu^i(e^i)}{\partial x^i} \qquad (40)$$

And

$$\frac{\partial \Psi_c^{jz,-i-1M,-i}}{\partial g_{DL}^{**zi}} = \frac{\partial \Psi_c^{jz,-i-JM,-i}}{\partial g_{DL}^{**zi}} = \frac{\partial \Psi_c^{jz,-i-1R,-i}}{\partial g_{DL}^{**zi}} = \frac{\partial \Psi_c^{jz,-i-1R,-i}}{\partial g_{DL}^{**zi}} = k^{-i}\frac{\partial \mu^{-i}(e^{-i})}{\partial g_{DL}^{zi}} \qquad (41)$$

Moreover, the convergence of the parties' policies implies

$$\frac{\partial F_c^{zi}}{\partial \Psi_c^{jzi-1Ri}} =.. \frac{\partial F_c^{zi}}{\partial \Psi_c^{jzi-JRi}} = \frac{\partial F_c^{z,-i}}{\partial \Psi_c^{jz,-i-1R,-i}} =.. \frac{\partial F_c^{z,-i}}{\partial \Psi_c^{jz,-i-1R,-i}} = f_c^{zi}(0) \in \mathbb{R}_+ \ \forall\, i, \forall\, z \qquad (42)$$

Use conditions (40) to (42) into equation (39) to show that $\forall g_{cL}^{**i} > 0,\ \forall i, \forall z$ satisfies

$$\int_{\forall\, e^i} h^i(e^i)\frac{\partial \mu^i}{\partial g_{cL}^{zi}} de^i + k^{-i}\int_{\forall\, e^{-i}} h^{-i}(e^{-i})\frac{\partial \mu^{-i}}{\partial g_{cL}^{zi}} de^{-i} = \left\{\frac{1}{N}\right\}\int_{\forall\, e^i} h^i(e^i)\frac{\partial \mu^i}{\partial x^i} de^i \qquad (43)$$

Lemma 4 says that parties in proportional representation systems with open party lists in a system of local governments select a policy platform on local public goods with and without spillovers that are Pareto efficient since parties have electoral incentives to recognize the whole distribution of benefits of local public goods provided by district i since the party seeks to maximize not only the votes in the local election of district i but also on the election of district $-i$. The expression in (37) says that parties select local public spending in district i where the marginal benefits of g_{DL}^{**zi} for residents of district i $\int_{\forall\, e^i} h^i(e^i)\frac{\partial \mu^i}{\partial g_{cL}^{zi}} de^i$ plus the externality on residents of district $-i$ of local public goods provided by district i, $k^{-i}\int_{\forall\, e^{-i}} h^{-i}(e^{-i})\frac{\partial \mu^{-i}}{\partial g_{cL}^{zi}} de^{-i}$ are equal to the marginal costs of providing g_{DL}^{**zi}, that is, $\left\{\frac{1}{N}\right\}\int_{\forall\, e^i} h^i(e^i)\frac{\partial \mu^i}{\partial x^i} de^i$.

Lemma 5 *The policy platforms chosen by a candidate j of party z in district i on economies with a proportional representation electoral system with open party lists, and a fiscally decentralized government are given as follows: Define $\eta_L^{jzi} \in [0,1]$ such that*

$$\eta_L^{jzi} = \frac{\frac{\partial Pr_{DL}^{jzi}}{\partial d^i}}{\frac{\partial Pr_{DL}^{jzi}}{\partial d^i} + \frac{\partial Pr_{DL}^{jzi}}{\partial \Psi_c^{jzi-1Mi}} + \cdots \frac{\partial Pr_{DL}^{jzi}}{\partial \Psi_c^{jzi-JMi}} + \frac{\partial Pr_{DL}^{jzi}}{\partial \Psi_c^{jzi-1Ri}} + \cdots \frac{\partial Pr_{DL}^{jzi}}{\partial \Psi_c^{jzi-JRi}}} \qquad (44)$$

Where $\frac{\partial Pr_{DL}^{jzi}}{\partial d^i}$ is the marginal effect of changes in d^i in the joint probability of candidate j of party z to obtain the nomination and a seat in the legislature of district i and the sum of $\frac{\partial Pr_{DL}^{jzi}}{\partial d^i}$ and $\frac{\partial Pr_{DL}^{jzi}}{\partial d^i} + \frac{\partial Pr_{DL}^{jzi}}{\partial \Psi_c^{jzi-1Mi}} + \cdots \frac{\partial Pr_{DL}^{jzi}}{\partial \Psi_c^{jzi-JMi}} + \frac{\partial Pr_{DL}^{jzi}}{\partial \Psi_c^{jzi-1Ri}} + \cdots \frac{\partial Pr_{DL}^{jzi}}{\partial \Psi_c^{jzi-JRi}}$ where the latter term is the marginal effect of candidate j of party z of obtaining a sufficient amount of votes to secure a seat in the legislature.

5.1) *If $\eta_L^{jzi} = 1$ then $g_{DL}^{**jzi} = g_{DL}^{**zi}$ which implies that local public goods with and without spillovers are Pareto efficient and given by*

$$\int_{\forall e^i} h^i(e^i) \frac{\partial \mu^i}{\partial G^i} de^i + k^{-i} \int_{\forall e^{-i}} h^{-i}(e^{-i}) \frac{\partial \mu^{-i}}{\partial G^{-i}} de^{-i} = \left\{\frac{1}{N}\right\} \int_{\forall e^i} h^i(e^i) \frac{\partial \mu^i}{\partial x^i} de^i \qquad (45)$$

5.2) *If $\eta_L^{jzi} = 0$ then local public goods are Pareto efficient only when local public goods do not show spillovers and are given by*

$$\int_{\forall e^i} h^i(e^i) \frac{\partial \mu^i}{\partial G^i} de^i = \left\{\frac{1}{N}\right\} \int_{\forall e^i} h^i(e^i) \frac{\partial \mu^i}{\partial x^i} de^i \quad \forall g_{DL}^{**jzi} > 0 \qquad (46)$$

Proof

In local election of district i a candidate j of party z proposes a platform of public spending g_{DL}^{**jzi} such that

$$g_{DL}^{**jzi} \in argmax\, Pr_{DL}^{jzi} \qquad (47)$$

Where

$$Pr_{DL}^{jzi} = Pr_{DL}^{jzi}\left(d^i, \left\{\Psi_c^{jzi-1Mi}\left(e^i\right) \dots \Psi_c^{jzi-JMi}\left(e^i\right)\right\}_{\forall e^i} \left\{\Psi_c^{jzi-1Ri}\left(e^i\right) \dots \Psi_c^{jzi-JRi}\left(e^i\right)\right\}_{\forall e^i}\right) \quad (48)$$

The first order condition is:

$$\frac{\partial Pr_{DL}^{jzi}}{\partial d}\frac{\partial d}{\partial g_{DL}^{jzi}} + \sum_{\forall e^i}\left\{\frac{\partial Pr_{DL}^{jzi}}{\partial \Psi_c^{jzi-1Mi}}\frac{\partial \Psi_c^{jzi-1Mi}\left(e^i\right)}{\partial g_{DL}^{jzi}} + \cdots \frac{\partial Pr_{DL}^{jzi}}{\partial \Psi_c^{jzi-JMi}}\frac{\partial \Psi_c^{jzi-JMi}\left(e^i\right)}{\partial g_{DL}^{jzi}}\right\}$$

$$\sum_{\forall e^i}\left\{\frac{\partial Pr_{DL}^{jzi}}{\partial \Psi_c^{jzi-1Ri}}\frac{\partial \Psi_c^{jzi-1Ri}\left(e^i\right)}{\partial g_{DL}^{jzi}} + \cdots \frac{\partial Pr_{DL}^{jzi}}{\partial \Psi_c^{jzi-JRi}}\frac{\partial \Psi_c^{jzi-JRi}\left(e^i\right)}{\partial g_{DL}^{jzi}}\right\} = 0 \quad \forall g_c^{**jzi} > 0 \qquad (49)$$

Moreover,

$$\frac{\partial \Psi_c^{jzi-1Mi}\left(e^i\right)}{\partial g_{DL}^{jzi}} \dots = \frac{\partial \Psi_c^{jzi-JMi}\left(e^i\right)}{\partial g_{DL}^{jzi}} =$$

$$= \frac{\partial \Psi_c^{jzi-1Ri}\left(e^i\right)}{\partial g_{DL}^{jzi}} \dots \frac{\partial \Psi_c^{jzi-JRi}\left(e^i\right)}{\partial g_{DL}^{jzi}} = \frac{\partial \mu^i\left(e^i\right)}{\partial g_{DL}^{jzi}} - \frac{1}{N}\frac{\partial \mu^i\left(e^i\right)}{\partial x^i} \qquad (50)$$

And

$$\frac{\partial d^i}{\partial g_{DL}^{jzi}} = \frac{g_{DL}^{jzi} - g_{DL}^{**zi}}{\left|g_{DL}^{jzi} - g_{DL}^{**zi}\right|} \qquad (51)$$

Define $\eta_L^{jzi} \in [0,1]$

$$\eta_L^{jzi} = \frac{\frac{\partial Pr_{DL}^{jzi}}{\partial d^i}}{\frac{\partial Pr_{DL}^{jzi}}{\partial d^i} + \frac{\partial Pr_{DL}^{jzi}}{\partial \Psi_c^{jzi-1Mi}} + \cdots \frac{\partial Pr_{DL}^{jzi}}{\partial \Psi_c^{jzi-JMi}} + \frac{\partial Pr_{DL}^{jzi}}{\partial \Psi_c^{jzi-1Ri}} + \cdots \frac{\partial Pr_{DL}^{jzi}}{\partial \Psi_c^{jzi-JRi}}} \qquad (52)$$

Express (49) as follows:

$$\eta_L^{jzi}\frac{g_{DL}^{jzi} - g_{DL}^{**zi}}{\left|g_{DL}^{jzi} - g_{DL}^{**zi}\right|} + \left\{1 - \eta_L^{jzi}\right\}\sum_{\forall e^i}\left\{\frac{\partial \mu^i\left(e^i\right)}{\partial g_{DL}^{jzi}} - \frac{1}{N}\frac{\partial \mu^i\left(e^i\right)}{\partial x^i}\right\} = 0 \quad \forall g_{DL}^{**jzi} > 0 \qquad (53)$$

Clearly if $\eta_L^{jzi} = 1$ then local public goods are Pareto efficient for local public goods with and without spillovers since $g_{DL}^{**jzi} = g_{DL}^{**zi}$ where

$$\int_{\forall e^i} h^i(e^i)\frac{\partial \mu^i}{\partial G^i} de^i + k^{-i} \int_{\forall e^{-i}} h^{-i}(e^{-i})\frac{\partial \mu^{-i}}{\partial G^{-i}} de^{-i} = \left\{\frac{1}{N}\right\} \int_{\forall e^i} h^i(e^i)\frac{\partial \mu^i}{\partial x^i} de^i \tag{54}$$

If $\eta^{jzi} = 0$ then $\sum_{\forall e^i}\left\{\frac{\partial \mu^i(e^i)}{\partial g_c^{jzi}} - \frac{1}{N}\frac{\partial \mu^i(e^i)}{\partial x^i}\right\} = 0 \quad \forall g_{DL}^{**jzi} > 0$ which implies

$$\int_{\forall e^i} h^i(e^i)\frac{\partial \mu^i}{\partial G^i} de^i = \left\{\frac{1}{N}\right\} \int_{\forall e^i} h^i(e^i)\frac{\partial \mu^i}{\partial x^i} de^i \quad \forall g_{DL}^{**jzi} > 0 \tag{55}$$

In economies with proportional representation and open party list systems, the candidates' policy platforms reflects a convex combination of two competing objectives: with a weight of $\eta_L^{jzi} > 0$ the candidate considers a policy that seeks to secure the party's nomination and with a weight $\{1 - \eta_L^{jzi}\} > 0$ the candidate seeks to maximize the expected number of votes in the local election of district i. if $\eta_L^{jzi} > 0$, candidates will select a policy platform that internalizes to some extent inter-regional spillovers. This issue will play a significant role in policy design of the local candidate as η_L^{jzi} approaches to one. For the limiting case in which $\eta_L^{jzi} = 1$ then the candidate of district i selects the ideal policy of the party. This policy maximizes the nationwide share of the vote of the party and the provision of local public goods with and without spillovers is Pareto efficient.

The weight the candidate assigns to the demands for local public spending of residents of district i plays a significant (minor) role in policy design if the value of η_L^{jzi} is close to zero (one). For the limiting case in which $\eta_L^{jzi} = 0$ then the candidate selects the ideal policy of the average voter in district i. By so doing, the candidate maximizes the expected proportion of the vote in the local election of district i which maximizes the candidate's chance to obtain a seat in the local legislature. In this case, the local public good in district i is Pareto efficient is local public goods do not show spillovers but it is not Pareto efficient for the case in which there is spillovers.

Lemma 6. *At the legislative stage of the local political process for an economy with a PR electoral system and open party lists, the provision of local public goods in each district* $i,-i$ *is given as follows:*

$6.i)$ *If* $\eta_L^{jzi} = 1$ *then* $g_{DL}^{***jzi} = g_{DL}^{**zi}$ *where*

$$\int_{\forall e^i} h^i(e^i)\frac{\partial \mu^i}{\partial G^i}de^i + k^{-i}\int_{\forall e^{-i}} h^{-i}(e^{-i})\frac{\partial \mu^{-i}}{\partial G^{-i}}de^{-i} = \left\{\frac{1}{N}\right\}\int_{\forall e^i} h^i(e^i)\frac{\partial \mu^i}{\partial x^i}de^i \tag{56}$$

$6.ii)$ *If* $\eta_L^{jzi} = 0$ *then* g_{DL}^{**jzi}*satisfy the following*

$$\int_{\forall e^i} h^i(e^i)\frac{\partial \mu^i}{\partial G^i}de^i = \left\{\frac{1}{N}\right\}\int_{\forall e^i} h^i(e^i)\frac{\partial \mu^i}{\partial x^i}de^i \quad \forall g_{DL}^{**jzi} > 0 \tag{57}$$

Proof

Since the policies of all candidates of all parties converge then the candidate's proposal will be approved by unanimity in the local legislature. Hence, if $\eta_L^{jzi} = 1$ then $g_{DL}^{***jzi} = g_{DL}^{**zi}$ which implies that local public goods with and without spillovers are Pareto efficient and given by

$$\int_{\forall e^i} h^i(e^i)\frac{\partial \mu^i}{\partial G^i}de^i + k^{-i}\int_{\forall e^{-i}} h^{-i}(e^{-i})\frac{\partial \mu^{-i}}{\partial G^{-i}}de^{-i} = \left\{\frac{1}{N}\right\}\int_{\forall e^i} h^i(e^i)\frac{\partial \mu^i}{\partial x^i}de^i \tag{58}$$

If $\eta_L^{jzi} = 0$ then local public goods are Pareto efficient only when local public goods do not show spillovers and are given by

$$\int_{\forall\, e^i} h^i(e^i)\frac{\partial\mu^i}{\partial G^i}de^i \;=\; \left\{\frac{1}{N}\right\}\int_{\forall\, e^i} h^i(e^i)\frac{\partial\mu^i}{\partial x^i}de^i \qquad \forall g_{DL}^{**jzi} > 0 \tag{60}$$

Theorem 2. Decentralization Theorem in PR Systems with Open Party Lists:

$T2.i$) If $\eta_L^{jzi} = 1\ \forall j, \forall z, \forall i$, the Strong and the Conventional Decentralization Theorem are satisfied. That is, in economies with PR electoral systems and open party lists, the provision of local public goods with and without inter-regional spillovers by a system of local governments welfare-dominates the fiscally centralized provision.

$T2.ii$) If $\eta_L^{jzi} = 0\ \forall j, \forall z, \forall i$, the Decentralization Theorem is satisfied. That is, in economies with PR electoral systems and open party lists, the provision of local public goods without inter-regional spillovers by a system of local governments welfare-dominates the fiscally centralized provision.

Proof

Again consider a nationwide social welfare function given by

$$\Psi(g^i, g^{-i}) = \sum_{\forall\, i,-i}\int_{\forall\, e^i} h^i(e^i)\, v^i(e^i, G^i)de^i \tag{61}$$

Assume Ψ is strictly concave. Let $\hat{\mathbf{g}}^* = [\hat{g}^{*i}, \hat{g}^{*,-i}]$ where $\hat{\mathbf{g}}^* \in argmax\ \Psi(g^i, g^{-i})$.

The first order condition for local public goods with inter-regional spillovers is given by

$$\int_{\forall\, e^i} h^i(e^i)\frac{\partial\mu^i}{\partial G^i}de^i + k^{-i}\int_{\forall\, e^{-i}} h^{-i}(e^{-i})\frac{\partial\mu^{-i}}{\partial G^{-i}}de^{-i} \;=\; \left\{\frac{1}{N}\right\}\int_{\forall\, e^i} h^i(e^i)\frac{\partial\mu^i}{\partial x^i}de^i \quad \forall\, \hat{g}^{*i} > 0 \tag{62}$$

For local public goods without inter-regional spillovers impose $k^{-i} = 0$ in condition (62).

Case 1: With $\boldsymbol{\eta^{jzi} = \eta_L^{jzi} = 1}\ \forall \boldsymbol{j}, \forall \boldsymbol{z}, \forall \boldsymbol{i}$:

The centralized provision of local public goods is given by $g_c^{***zi} = g_c^{**zi}\ \forall i$, satisfying

$$\sum_{\forall\, i,-i}\left(1+k^{-i}\right)\int_{\forall\, e^{i}} h^{i}\left(e^{i}\right)\frac{\partial\mu^{i}}{\partial G^{i}}de^{i}=\left\{\frac{1}{N}\right\}\sum_{\forall i,-\, i}\int_{\forall\, e^{i}} h^{i}\left(e^{i}\right)\frac{\partial\mu^{i}}{\partial x^{i}}de^{i}0 \quad \forall g_{c}^{***jzi}>0 \quad (63)$$

The decentralized provision of local public goods is given *by* $g_{DL}^{***zi}=g_{DL}^{**zi}\ \forall i$ *satisfying*

$$\int_{\forall\, e^{i}} h^{i}\left(e^{i}\right)\frac{\partial\mu^{i}}{\partial G^{i}}de^{i}+k^{-i}\int_{\forall\, e^{-i}} h^{-i}\left(e^{-i}\right)\frac{\partial\mu^{-i}}{\partial G^{-i}}de^{-i}=\left\{\frac{1}{N}\right\}\int_{\forall\, e^{i}} h^{i}\left(e^{i}\right)\frac{\partial\mu^{i}}{\partial x^{i}}de^{i} \quad (64)$$

Clearly $g_{DL}^{**zi}=\hat{g}^{*i}\ \forall i$ and $g_{c}^{**zi}\neq\hat{g}^{*i}$. Since Ψ is strictly concave then the solution to the nationwide social welfare function is unique then

$$\Psi\left(g_{DL}^{***zi},g_{DL}^{***z,-i}\right)>\Psi\left(g_{c}^{***zi},g_{c}^{***z,-i}\right) \quad (65)$$

Condition (65) shows that the decentralized provision of local public goods with spillovers welfare dominates the centralized provision.

Case 2: With $\boldsymbol{\eta^{jzi}=\eta_{L}^{jzi}=0\ \forall j,\forall z,\forall i}$:

The centralized provision of local public goods is given by $g_{c}^{***zi}=g_{c}^{**zi}\ \forall i,$, satisfying

$$\int_{\forall\, e^{i}} h^{i}\left(e^{i}\right)\frac{\partial\mu^{i}}{\partial G^{i}}de^{i}=\int_{\forall\, e^{i}} h^{i}\left(e^{i}\right)\frac{\partial\mu^{i}}{\partial x^{i}}de^{i} \qquad \forall g_{c}^{***jzi}>0 \quad (66)$$

The decentralized provision of local public goods is given *by* $g_{DL}^{***zi}=g_{DL}^{**zi}\ \forall i$ *satisfying*

$$\int_{\forall\, e^{i}} h^{i}\left(e^{i}\right)\frac{\partial\mu^{i}}{\partial G^{i}}de^{i}+k^{-i}\int_{\forall\, e^{-i}} h^{-i}\left(e^{-i}\right)\frac{\partial\mu^{-i}}{\partial G^{-i}}de^{-i}=\left\{\frac{1}{N}\right\}\int_{\forall\, e^{i}} h^{i}\left(e^{i}\right)\frac{\partial\mu^{i}}{\partial x^{i}}de^{i} \quad (67)$$

Clearly $g_{DL}^{***zi}=\hat{g}^{*i}\ \forall i$ and $g_{c}^{***zi}\neq\hat{g}^{*i}$. Since Ψ is strictly concave then the solution to the nationwide social welfare function is unique which implies

$$\Psi\left(g_{DL}^{***zi},g_{DL}^{***z,-i}\right)>\Psi\left(g_{c}^{***zi},g_{c}^{***z,-i}\right) \quad (68)$$

Condition (68) shows that the decentralized provision of local public goods without spillovers welfare dominates the centralized provision.

Theorem 2 shows that in economies with proportional representation electoral systems and open party lists, the strong and conventional decentralization theorem are satisfied if $\eta_L^{jzi} = 1\ \forall j, \forall z, \forall i$ and only the conventional decentralization theorem is satisfied if $\eta_L^{jzi} = 0\ \forall j, \forall z, \forall i$. In our economy, candidates seek to get the nomination of their parties and the maximum amount possible of votes from the electorate to guarantee a seat in the legislature. If $\eta_L^{jzi} = 1\ \forall j, \forall z, \forall i$ in the economy there is *party centralization*, nomination concerns induce candidates to adopt the policy platform of the party which seeks to maximize the nationwide proportion of the vote. Since parties face multiple local electoral contests, they have incentives to internalize the spillovers of the local public good of district i in other districts. As a result, the fiscally decentralized provision of local public goods with and without spillovers is Pareto efficient.

A nationwide election and party centralization (again if $\eta_L^{jzi} = 1\ \forall j, \forall z, \forall i$) create incentives for candidates of parties to provide local public goods with and without spillovers in a Pareto efficient way. In this case, any party $z = \{L, M, R\}$ seeks to maximize the nationwide proportion of the vote and therefore the party has incentives to incorporate the fact that spillovers of local public goods of district i affect the probability of residents of other districts to vote for the party in the nationwide election. Hence, the fiscally centralized provision of local public goods with and without spillovers is also Pareto efficient.

Moreover, since a system of local governments can differentiate local public goods to maximize the gains associated with matching policy with the heterogeneous demands of individuals for local public spending across districts while the central government is constrained to provide uniform local public goods, therefore the fiscally decentralized structure of government welfare dominates the fiscally centralized structure.

The other case of interest is when $\eta_L^{jzi} = 0\ \forall j, \forall z, \forall i$. In this case, there is *party decentralization*, and nomination concerns do not play a significant influence in the design of the candidate's policy platform. For the case of a system of local governments, candidates seek to maximize their expected proportion of the vote in the local election of district i to maximize the candidates' chance to obtain a seat in the local legislature. In this case, candidates select the ideal policy of the average voter of district i and therefore the fiscally decentralized provision is Pareto efficient when local public goods do not show spillovers but it is not Pareto efficient if local public goods have spillovers.

Fir the case of fiscal centralization, a nationwide election and party decentralization (again if $\eta^{jzi} = 0\ \forall j, \forall z, \forall i$) create incentives for candidates to select a policy platform that maximizes the number of votes from residents.[14] As it is shown in Lemma 3, the weakly dominant strategy for candidate j of party z in district i is to select the ideal policy of the average voter of districts i to secure the highest number of votes from the electorate in district i. However, for the case of $\eta^{jzi} = 0\ \forall j, \forall z, \forall i$, candidates of parties in each district do not recognize the external benefits on other districts of local public goods provided by district i. Therefore, local public goods are Pareto efficient only when local public goods do not have spillovers.

As before, since a system of local governments can differentiate local public goods without spillovers to maximize the gains associated with matching policy with the heterogeneous demands for local public spending across districts while the central government is constrained to provide uniform local public goods then the fiscally decentralized structure of government welfare dominates the fiscally centralized structure. The conventional decentralization theorem is

[14] Recall that the nationwide legislature is constituted by representatives chosen in each district.

satisfied. However, the strong decentralization theorem is not satisfied since neither the system of local governments nor the central government provide local public goods with spillovers that are Pareto efficient.

VI Conclusion

In this paper we analyze whether the decentralization theorem, advanced by Oates (1972,1995), is satisfied or not for economies with a proportional electoral systems with closed and open party lists. The decentralization theorem is at the core of the fiscal federalism literature. However, it is not evident if the fiscally decentralized provision of local public goods is superior to the centralized provision once we remove Oates' assumption that governments are ruled by benevolent social planners and incorporate electoral competition and some specific institutions of the country related to the electoral system. In this paper we seek to contribute to the literature by studying the welfare properties of the fiscally (de)centralized provision for economies with proportional electoral systems with closed and open party lists.

We find that the institutional set up of proportional representation systems matters for the welfare properties related with the provision of local public goods. In particular, a strong and the conventional decentralization theorems hold in proportional systems with closed party lists. The strong decentralization theorem says that the provision of local public goods *with inter-regional spillovers* by a system of local governments (welfare) dominates the fiscally centralized provision in economies with proportional representation and closed party list electoral systems. In addition, the conventional decentralization theorem is satisfied implying that the provision of local public goods *without inter-regional spillovers* by a system of local governments welfare-dominates the fiscally centralized provision in economies with proportional representation and closed party list electoral systems.

For economies with proportional representation and open party lists systems the strong and the conventional decentralization theorem are satisfied only under certain conditions. In this economy, candidates announce a binding policy platform that maximizes the joint probability of obtaining the nomination of the party in the list and the highest number of votes from the electorate to secure a high position in the party's list that might lead the candidate to hold a seat in the legislature. Candidates therefore face conflicting objectives in selecting policy. On the one hand, candidates would like to adopt the parties' policy platform to secure the party's nomination. On the other hand, candidates also need to appeal to the electorate to guarantee a high number of votes and a seat in the legislature. In this context, if there is party centralization (i.e., if the party's power to nominate candidates to the open list exert a significant role in determining the policy of candidates), the provision of local public goods with and without inter-regional spillovers by a system of local governments in economies with proportional representation and open party list electoral systems welfare-dominates the fiscally centralized provision. In this case, the strong and the conventional decentralization theorem are satisfied.

However, if there is party decentralization (the party's power to nominate candidates to the open list does not exert a significant role in determining the policy platforms of candidates) only the conventional decentralization theorem is satisfied. That is, the provision of local public goods without inter-regional spillovers by a system of local governments in economies with PR and open party list electoral systems welfare-dominates the fiscally centralized provision when the party system is centralized. However, the strong decentralization theorem is not satisfied.

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