

International Center for Public Policy
Working Paper 24-17
December 2024

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Reconciling Tax Buoyancy and Tax Capacity

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Abstract

I attempt to reconcile two vast strands of literature that essentially estimate the same empirical relationship. Tax effort studies aim to benchmark a country's tax-to-GDP ratio to tax outcomes observed in other countries under comparable conditions, in particular under similar levels of economic development, proxied with the real GDP per capita. A completely separate strand of literature deals with estimating tax buoyancy, which is measured as the percentage change in tax revenue associated with a 1-percent change in GDP. While dealing with some of the same data (tax revenues and GDP) as in the tax effort studies, the tax buoyancy literature has developed econometric methods that are more robust to the empirical challenges presented by these data. In this paper, I establish correspondence between the statistical parameters estimated in these two separate strands of literature. Thus, I show that an estimate of long-run buoyancy can be translated into the magnitude of the impact of economic development on the tax-to-GDP ratio by making adjustments for how the population size and real exchange rate interact with economic growth.

Keywords: tax buoyancy, tax compliance, tax policy, GDP

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I am grateful for helpful comments from participants at the 36th Annual Conference of the Association for Budgeting and Financial Management in Cleveland, OH on September 26–28, 2024.

Introduction

In this paper I attempt to reconcile two strands of literature, which have been developing for over seven decades, largely in isolation from each other, while essentially attempting to estimate the same empirical relation. Tax effort studies aim to produce a revenue performance metric for a country by benchmarking its tax-to-GDP ratio to tax outcomes observed in other countries under comparable conditions. However, unlike typical benchmarking, tax effort studies go one step further by tailoring this benchmark specifically to the country of interest according to the country's characteristics. Then, a Tax Effort indicator is calculated as the ratio of the actual tax collection over the benchmark value (Tax Capacity) estimated for a given country. Thus, this benchmark is not obtained by just averaging tax outcomes in other countries but instead synthesized by making adjustments for differences in country characteristics.

In the early tax effort studies, the level of economic development, proxied with the real GDP per capita, was the only dimension used to adjust the tax capacity benchmark. Subsequent studies (e.g., Hinrichs 1965) attempted to replace economic development with more granular explanatory variables, such as trade openness, which would better capture hypothesized causal channels approximated with GDP per capita. However, these studies found the level of economic development still retaining some explanatory power, even after the inclusion of those more granular determinants of tax outcomes (Tanzi and McCuistion 1967).

An entirely separate strand of literature deals with estimating tax buoyancy, which is measured as the percentage change in tax revenue associated with a 1-percent change in the national income. In practice, estimates of tax buoyancy have become instrumental for the calculation of cyclically adjusted and structural budget balances as part of fiscal surveillance, for example in

the implementation of fiscal rules established in the European Union under the Stability and Growth Pact of 1997 (Koester and Priesmeier 2017).

While dealing with some of the same data (tax revenues and GDP) as in the tax effort studies, the tax buoyancy literature has developed econometric methods more robust to the empirical challenges presented by these data: slowly evolving dependent and independent variables, endogeneity of GDP and other determinants of tax outcomes, cross-section dependences (CSD) due to unobservable shocks common to all countries, heterogeneity of country slopes, measurement errors, etc. Furthermore, the methodology used in the tax buoyancy studies allows one to obtain separate estimates for short-run buoyancy, relevant for economic stabilization, and long-run buoyancy, relevant for fiscal sustainability.

In this paper, I establish correspondence between the statistical parameters estimated in these two strands of literature. In particular, I show that an estimate of long-run buoyancy can be translated into the magnitude of the impact of economic development on the tax-to-GDP ratio by making adjustments for how the population size and real exchange rate interact with economic growth. Among other things, this correspondence allows one to use estimates produced with more robust methods in the tax buoyancy literature in order to validate the parameters estimated to produce the tax capacity benchmark in the tax effort studies. I also attempt to draw on the recent advances in the tax buoyancy literature to provide some methodological guidance for tax effort studies.

Literature

Tax buoyancy studies

The origins of the tax buoyancy literature can be traced back to the discussions of countercyclical policies, and in particular to the notion of “automatic stabilizers” and associated empirical estimates of “built-in flexibility” of the federal tax system in the United States

(Musgrave and Miller 1948). Those discussions were primarily concerned with the *stability of the national income* and therefore utilized macroeconomic data when modeling “automatically compensatory movement of [federal] tax revenues” (p. 122).

Building on those earlier studies, Groves and Kahn (1952) instead focused on the *stability of tax revenues* and employed state and local data. They argued that, rather than macroeconomic stability pursued at the federal level, among other things with a proper design of the federal tax system, subnational levels of government are more concerned with “maintaining a given level of tax-financed government services” (p. 88). While not yet allowing for asymmetric responses of tax revenues to economic booms and busts, Groves and Kahn did discuss different implications for sustainability of government services associated with the same degree of elasticity in the deflationary versus inflationary environment, which was still topical given the memories of the Great Depression (captured in some of the data they used). They proposed decomposition of the income elasticity of taxes into two separate components: one related to changes in real income while the other arising from price changes.

Groves and Kahn (1952) used data only on some taxes and in some states “to avoid situations involving major changes other than those in income.” Subsequent literature distinguished tax buoyancy (observed changes in tax revenues including discretionary tax changes) from “built-in tax elasticity” (Mansfield 1972, p. 425).

While this earlier literature was insightful, their empirical methods were not adequate for the time-series nature of the data they were using. Sobel and Holcombe (1996) later pointed out the importance of distinguishing between “short-run variability and long-run growth” and suggested that, for the same dependent variable, one can obtain two very different estimates of short- and long-run buoyancy, respectively.

Tax effort studies

There have been multiple iterations of tax effort studies conducted by the IMF over the last 50 years. Not long before the first IMF study, the notions of “fiscal capacity” and “tax effort” originally emerged in the application to subnational government units in the field of intergovernmental fiscal relations (e.g., ACIR 1962). Earlier studies of tax effort (e.g., Lotz and Morss 1967, Chelliah 1971, Bahl 1971), while acknowledging a number of observable determinants of tax effort, chose not to include those variables as explanatory variables, leaving them in the regression residual. There has been also a lot of cross-fertilization between tax effort studies and positive empirical studies explaining differences in tax systems across countries (e.g., Williamson 1961, Hinrichs 1965, Thorn 1967, Shin 1969, Weiss 1969).

The relevance of tax elasticity was acknowledged in some tax effort studies without any mentioning of the tax buoyancy literature cited above. This was first articulated by Chelliah (1971, p. 301):

Strictly speaking, tax effort is a process; it takes several forms, including reform of existing taxes, improvement in administration, and introduction of new taxes. All these steps necessarily require time to plan, legislate, and implement. Countries that for historical or other reasons started out with a low tax ratio a decade or so ago might have undertaken considerable effort to raise their respective tax ratios but may not yet have reached even the average level of taxes in developing countries. Tax effort, therefore, should be considered also in the dynamic sense of comparing changes in the tax ratio over time. Thus, even if a country has a low tax effort in the static sense, the question may be asked whether it has made efforts over a period of time to increase tax revenues. For this purpose, it seems best (for reasons explained earlier) to compare the income elasticities of total taxes.

Later, Newlan (1985) proposed measuring tax effort by using an estimate of revenue elasticity as a benchmark for revenue buoyancy. Indeed, the relation between tax capacity (unobserved) and the tax-to-GDP ratio (observed) is similar to the relation between revenue elasticity (counterfactual tax revenue change holding the tax policy constant) and revenue buoyancy (actual tax revenue change including discretionary policy interventions). However, revenue elasticity estimates make adjustments only for tax policy efforts while neglecting tax administration efforts. In a dynamic setting, a distinction should also be made between long-term elasticity (equilibrium) and short-term deviations/adjustments.

Unit income elasticity of taxes

In the tax buoyancy literature, estimates of income elasticity of tax revenues (i.e., buoyancy) are often compared to unity in order to evaluate fiscal sustainability in a given country.

Groves and Kahn (1952, p. 88) believed that the cost of (subnational) government services remained constant in real terms and argued that "...[real] tax yields could remain constant and still sustain government service." Thus, they argued that nominal tax revenues should only respond to price changes but not to changes in real output so that the elasticity with respect to the nominal income should be less than one. In their words, "...if it is desired to maintain intact at all times the level of government services and to finance them from taxes primarily, the total of state and local tax revenues has to be of less than unit income elasticity."

Belinga et al. (2014, p. 5) argued that above-unity, long-run buoyancy would "ceteris paribus imply that higher growth will improve the fiscal balance through the revenue side." Koester and Priesmeier (2017, p. 5) benchmarked their estimates of long-run buoyancy against unity, which they declared "consistent with a constant revenue to GDP ratio in the long term." Dudine and Jalles (2018, p. 962) argued that "a buoyancy greater than unity over the long run is a desirable

feature of a tax system if there is increasing demand for public services and if a country would like to pursue relative financial stability.”

By contrast, Lagravinese et al. (2020, p. 4) argued that the long-run buoyancy has to be unity in order for public finances to be sustainable. As they put it,

Indeed, a value greater than 1 would imply that the ratio of taxes to GDP would increase indefinitely; while a value lower than 1 would mean that the same ratio would fall continuously. Both cases do not represent a long-run equilibrium for the sustainability of public finances.

The latter statement might be technically correct if the underlying relationship indeed exhibited a constant elasticity rather than being approximated with a log-linear relationship for analytical convenience. However, if the elasticity of the underlying relationship were not constant but, say, decreasing with economic development, then the tax-to-GDP ratio could be continually increasing while approaching its long-run limit from below. Functional forms that do not exhibit a constant elasticity are less tractable, but some suggestions have been made in the tax effort literature, albeit not yet explored, such as a hyperbolic function (Williamson 1961). In the tax effort literature, Mawejje and Sebudde (2019), Fenchietto and Pessino (2013) and Langford and Ohlenburg (2016) allowed for a non-linear impact of economic development by including a quadratic term of the log of per capita GDP. However, this would imply that, at some level of economic development, the tax-to-GDP ratio would start declining rather than continually approaching its long-run limit from below.

Empirical Framework

As already mentioned, the level of economic development is not the only determinant of tax capacity. Some determinants of tax outcomes are not captured in empirical models because they

are hard to measure (e.g., culture) or too numerous (e.g., indirect determinants). Omitting a variable that is a determinant of the dependent variable can lead to a biased estimate of the impacts of the included explanatory variables that are correlated with that omitted variable. This so-called omitted variable bias is a special case of the endogeneity problem, which also includes a simultaneity bias, discussed below.

Repeated observations on the same country allow one to account for the impact of those “unobservable” factors, especially if they change slowly over time (or do not change at all). In addition, using more observations leads to more precise estimates. However, with panel data comes a range of additional assumptions that one needs to make in modeling the relationships across counties and time periods. Different assumptions made in specifying the model can lead to dramatic differences in the estimates of tax capacity and effort.

In the most general form, a panel data model is specified as

$$y_{it} = m(x_{it}; \beta_i) + c_i + \eta_i + u_{it} + v_{it} \quad (1)$$

- with data indexed by country, i , and time, t ,
- y_{it} measuring tax outcomes (e.g., tax-to-GDP)
- $m(x_{it}; \beta)$ capturing the impact of observed country characteristics x_{it}
- c_i capturing time-invariant unobserved differences in tax capacity
- v_{it} capturing stochastic shocks (impact of all determinants of tax capacity other than those captured by the measurable country characteristics x_{it})
- η_i encapsulating time-invariant (persistent) unobserved differences in tax effort
- u_{it} representing time-varying (transient) tax effort.

Until very recently, there were no methods available to estimate a model in this general form, in particular capable of disentangling all four unobservable components of the disturbance term.

Typically, a tax buoyancy estimate is obtained as a regression coefficient on the nominal GDP from estimating equation (1) in a double-logarithmic functional form with a log of nominal tax revenues as the dependent variable. In a rare departure, Adam et al. (2001) derived tax buoyancy

as $1+b$, where b is an estimated coefficient from regressing the tax-to-GDP ratio on the log of real GDP per capita.

In tax effort studies, equation (1) is estimated with the tax-to-GDP ratio on the left-hand side and real GDP per capita on the right hand side, but the functional form varies between studies: double-logarithmic (e.g., Teera and Hudson 2004), double-logarithmic with an additional quadratic term of the log of real GDP per capita (Fenochietto and Pessino 2013; Mawejje and Sebudde 2019), single-logarithmic (Ghura 1998; Gupta 2007). Lotz and Morss (1967) used of a quadratic rather than a logarithmic function.

Empirical challenges

The nature of the hypothesized determinants of tax outcomes presents a number of challenges when using standard statistical methods for estimating the impact of these determinants, in particular the level of economic development, on the observed tax outcomes. Immediately below, I lay out and discuss the main methodological challenges.

Simultaneity

GDP, foreign aid, indebtedness, and other explanatory variables can be jointly determined with the tax outcomes. Thus, while tax capacity has been hypothesized to increase with the level of economic development, at the same time taxation can affect economic development. Thus, the observed international experiences might represent equilibria that allow countries to maintain a certain level of taxation along with a certain level of economic development. Such simultaneity complicates the estimation of the statistical parameters quantifying this relationship, as it violates the assumption of strict exogeneity of the explanatory variables typical in standard statistical methods. In a situation when this assumption is violated, one has to deal with endogeneity,

meaning that an explanatory variable is correlated with some components of the disturbance term ($c_i + \eta_i + u_{it} + v_{it}$) in the regression model.

If endogeneity cannot be eliminated, for example by including omitted variables, a common solution is to find auxiliary variables, called instrumental variables or simply instruments. These instrumental variables are used in regression analysis as tools to isolate the components of explanatory variables that are uncorrelated with the disturbance term.

When not using such instrumental variables, an estimate obtained with a standard statistical method would represent a net effect (a “reduced form”) of such simultaneous determination rather than a causal effect that would quantify an impact of a change in the explanatory variable (such as economic development), while holding the reverse effect (from taxation to development) constant. While not suitable for impact evaluation of policy interventions, such reduced form estimates can still be useful for forecasting, which is one of the applications of tax buoyancy.

Slowly evolving institutional variables

Another problem with the data on tax outcomes and their determinants is that these are slowly evolving variables with past shocks fading out slowly over a number of subsequent periods. This persistence of variables violates a common assumption of standard statistical methods, which require independently distributed disturbances. This violation can lead to misleading inferences due to autocorrelation of the disturbances. Thus, empirical studies have found some of the macroeconomic explanatory variables used in tax effort regressions, such as GDP per capita, to be non-stationary, meaning that they had a persistent trend, or the effects of past shocks never went away.

Furthermore, when shocks persist over time periods, in addition to the instantaneous (short-run) impact of any determinant on tax outcomes in one time period, there could be additional delayed impacts in subsequent periods. This brings the notion of a long-run impact, which captures the cumulative impact across time periods, both instantaneous and delayed.

The issue of persistent variables takes an extreme form in the case of non-stationarity, meaning that the distribution of future values of some variable does not look like the distribution of the past values of the same variable. This can happen when past shocks never fade away, no matter how many periods of time have passed. When data are non-stationary, then outputs from standard statistical methods (goodness of fit or R-squared, standard errors and significance levels of estimates and hypotheses testing) can become unreliable.

Common shocks (cross-section dependence)

In addition to persistence over time, the assumption of independently distributed disturbances can be also violated by shocks correlated across countries (e.g., a pandemic affecting all countries at the same time).

Spatial dependence

Furthermore, the impact of common shocks (e.g., weather events) can vary with the proximity of countries to each other in terms of their geography, trade, etc.

Measurement errors

Measurement errors are inevitable, as our explanatory variables are imperfect proxies for socio-economic processes we are trying to account for (e.g., economic development proxied with GDP per capita). Furthermore, these measurement errors may dominate annual changes in slowly evolving institutional characteristics we are trying to measure.

Country slope heterogeneity

Even if the hypothesized determinants of tax outcomes are relevant for each country, the quantitative impact of these determinants might not be the same in each country. Standard methods of panel data analysis assume that slope coefficients are the same for each country. When these standard methods are applied to data coming from a process where countries have different slope coefficients, the estimated uniform slope coefficient might not be even converging to an “average” of country-specific slope coefficients, even as the sample of countries increases. That is, the law of large numbers might not work for “averaging” in this case. Estimating an “average” coefficient in the case of heterogeneous panels, where each country can have a different slope, is the essence of the tax capacity concept, which is supposed to capture average or “standard” relationships between countries’ characteristics and their tax outcomes (Bahl 1971). Furthermore, slope heterogeneity can cause a separate kind of lack of convergence (inconsistency) in models using lagged values of variables to account for the persistence of past shocks. This second type of inconsistency persists even if the dataset covers a large number of time periods (Pesaran and Smith 1995).

Robustness of alternative estimators

Having enumerated various empirical challenges posed by our data, next I discuss to what extent alternative estimation strategies are robust to these challenges.

Between estimators

When using panel data, there are two ways to look at the relationships among variables in such a dataset. First, one can examine how cross-country differences in tax-to-GDP ratios are linked to differences in country characteristics, which change very slowly over time, if at all. This is achieved with the help of the “between estimator,” which essentially takes the mean of each

variable for each country over time and runs a regression on the collapsed dataset of means. This strategy allows one to examine the impact of the country's characteristics discussed above, which, while changing slowly over time, vary more significantly across countries. However, the collapsing of data results in a loss of information, in particular on the evolution of these variables over time.

Earlier studies of tax effort, used three- or five-year averages “in order to minimize the influence of fortuitous factors.” (Chelliah 1971, p. 256, Lotz and Morss 1967, Bahl 1971, Weiss 1969, Tait et al. 1979). Time averaging of the actual tax outcomes was supposed to eliminate transient shocks to tax capacity (e.g., cyclical fluctuations) so that the remaining deviations from the estimated average relationship can be attributed to tax effort. For panels with a sufficiently large number of time periods, this strategy found some validation in the econometric theory. Under certain assumptions, when the number of time periods T and countries N are large, such a cross-section regression based on the time averages of variables should produce a consistent estimate of the mean of the long-run coefficients (Pesaran and Smith 1995).

Static (“within”) estimators

In contrast to the between estimator, one can focus on changes of these variables over time while sweeping out the effects of time-invariant factors, in particular those that I cannot observe directly. However, two kinds of problems arise when this so-called “within” approach is pursued through the common techniques of first-differencing or demeaning achieved with the inclusion of country dummies as the so-called “fixed effects” (e.g., Teera and Hudson 2004 and Le et al. 2012). First, short-term fluctuations of macro-fiscal variables—for example, lower revenues from income taxes in times of recession—would be irrelevant for our study of tax effort, which should be considered over the whole business cycle and, like other policies, fully play out only in

the medium term. Second, under both first-differencing and fixed-effect techniques, the measurement error problem can be exacerbated (Griliches and Hausman 1986).

Dynamic estimators

One can attempt to account for the persistence (autocorrelation) of the dependent variable by including its lagged values as additional regressors. However, in the presence of unobserved time-invariant heterogeneity (“fixed effects”), this can add a downward bias to the estimates when the number of time periods (T) is small (Nickell 1981). Furthermore, slope heterogeneity can cause a separate kind of inconsistency in models with lagged variables, which persist even for large T (Pesaran and Smith 1995).

When the slopes are the same for all countries, one solution to both the measurement errors¹ and autocorrelation of disturbances is to use instrumental variable estimators proposed under the so-called generalized method of moments (GMM) approach, which allows for a large number of weak instruments (e.g., Gupta 2007). Additionally, using instrumental variables allows parameters to be estimated consistently even in the case of simultaneity. Under the difference GMM estimator proposed by Arellano and Bond (1991), lagged levels are used as instruments for subsequent first differences. However, the difference GMM estimator does not perform very well with a small number of time periods, especially when the persistence of past shocks is high and the variation in country fixed effects is large relative to the variation of the residual white-noise disturbances (Blundell and Bond 1998, Kiviet et al. 2017).

When the number of time periods in the dataset is sufficiently large, additional estimation strategies are possible, in particular those allowing heterogeneity in slopes as opposed to just

¹ When the measurement errors are autocorrelated rather than being white noise, the difference GMM estimator does not eliminate the measurement error bias (Hauk and Wacziarg 2009).

heterogeneity in intercepts featured in the “within” regressions. When the cross-section dimension (N) is small (e.g., $N < 10$) and the time series dimension (T) is large, one could treat the set of equations from individual countries as a system of seemingly unrelated regression equations (SURE), thus allowing for general (time-invariant) correlation patterns across the errors in the different cross-section equations. Then, one could compute the mean of the estimates from these individual equations to produce what was called the mean group (MG) estimator by Pesaran et al. (1999). However, if the number of countries (N) were allowed to rise, the application of the SURE approach would involve nuisance parameters whose number would increase at a quadratic rate. This would make SURE not feasible when N were large relative to T , as the estimated covariance matrix could not be inverted. Furthermore, the SURE approach is not applicable when the disturbance terms are correlated with the regressors.

The Pooled Mean Group (PMG) estimator was initially proposed by Pesaran et al. (1999) for models with lagged dependent variables and heterogeneity of short-run dynamics while constraining the long-run coefficients to be the same for all units (i.e., all countries in our case). Furthermore, the initially proposed PMG estimator did not allow for error cross-section dependence at that time.

To deal with the latter problem, a new approach was proposed by Pesaran (2006) leading to a set of estimators known as common correlated effects (CCE) estimators. This approach aims to eliminate the differential effects of unobserved common factors by filtering the country-specific regressors by means of cross-section (weighted) averages of the dependent variable and the observed country-specific regressors. While Pesaran (2006) initially developed this approach for the case of stationary and exogenous country-specific regressors and common factors, it was

subsequently extended to the case where the unobserved common factors were allowed to follow unit-root processes (Kapetanios et al. 2011).

The CCE approach also offers an empirical framework for estimating an “average” coefficient in the case of heterogeneous panels where each country can have a different slope. Building on the Mean Group (MG) estimator initially proposed in Pesaran and Smith (1995), the “Common Correlated Effects Mean Group” (CCEMG) estimator is a simple average of the CCE estimators obtained for individual countries. An alternative estimator— called the “Common Correlated Effects Pooled” (CCEP) estimator—is a generalization of the fixed effects estimator, which imposes uniformity of country slopes but now allowing cross-section dependence.

The initial CCE approach allowed for dynamics through autocorrelation both in the common factors (even unit-root processes as shown later in Kapetanios et al. 2011) and in country-specific effects and regressors. However, initially, it did not explicitly model these dynamics by including the lagged values of the dependent variable among the regressors. The assumption of strictly exogenous regressors in Pesaran (2006) not only did not allow the lagged values of the dependent variable among the regressors, but it also ruled out feedback effects from the dependent variable onto the regressors—for example, from the level of taxation to GDP in the context of this study.

Chudik and Pesaran (2015) extended the CCE approach by introducing lagged dependent variables and allowing regressors to correlate with past disturbances (weak exogeneity). Such dynamic CCE (DCCE) estimators open new possibilities for studying tax outcomes using data panels with a sufficiently large number of time periods (T). With such data at hand, one could improve on the Mean Group (MG) and Pooled Mean Group (PMG) estimators, which did not allow for error cross-section dependence. With the dynamic CCE extension, the Mean Group

estimator (DCCE-MG) could feature a large degree of heterogeneity, by allowing both the intercepts and the slopes (e.g., the impact of economic development) to vary across countries, both in the short and long run. Thus, one could potentially test whether there is a uniform long-run relationship between the tax-to-GDP ratio and the level of economic development.

Table 1. Robustness of Alternative Estimators

Estimator / robust to	Slope heterogeneity	Measurement error	Simultaneity	Autocorrelation	Cross-section dependence
Between	Yes	Somewhat	No	Yes	Yes
Fixed effects	No	No	No	No	No
Error Components	No	No	No	No	No
Frontier	No	No	No	No	No
Difference	No	No*	Yes	Yes	No
GMM	No	No*	Yes	Yes	No
CCE-MG	Yes	?	No	Yes	Yes
DCCE-MG	Yes	?	Yes	Yes	Yes

Source: Timofeev (2023)

Notes: * When the measurement errors are autocorrelated rather than being white noise, the difference GMM estimator does not eliminate the measurement error bias.

Table 1 summarizes the preceding discussion of the robustness of the reviewed econometric methods to various empirical challenges posed by the data used in tax effort studies.

Tax buoyancy estimates

Using data on a panel of 34 OECD countries between 1965–2012, Belinga et al. (2014) found above unitary long-run buoyancy of the aggregate tax revenues—inclusive of social security contributions (SSC)—in 14 countries, below unitary long-run buoyancy in four countries,² while it was found not statistically significantly different from unity in the remaining 16 countries. They also produced a pooled mean group (PMG) estimator for a balanced panel of 22 countries. For the entire 1965–2012 period, the PMG estimator of the long-run buoyancy was 1.06 and

² Hungary, Israel, Netherlands, and Slovak Republic.

statistically significantly different from unity. However, it was estimated to be only 1.02 and *not* statistically significantly different from unity on the 1989–2012 subsample, while 1.09 and statistically significantly different from unity on the 1965–1988 subsample.

When analyzing data on 25 OECD countries in 1965–2015, Deli et al. (2018) obtained estimates of long-run buoyancy not statistically different from unity for all but nine countries. In their dynamic panel analysis, the long-run buoyancy of total tax revenues was estimated to be 1.03 using the Mean Group (MG) estimator and 1.006 using the PMG estimator. Neither estimate was statistically different from unity.

For a shorter but broader panel of 35 OECD countries over 1995–2016, Lagravinese et al. (2020) obtained an estimate equal to 0.91 for long-run tax buoyancy of total tax revenues, inclusive of SSC, using the Dynamic Common Correlated Effects estimator (DCCE). Furthermore, they found this estimate to be statistically significantly different than unity.

While applying the fully modified OLS (FMOLS, proposed by Phillips and Hansen 1990) to the time series for each of 107 countries over 1970–2014, Dudine and Jalles (2018) found the average estimate of long-run buoyancy of total tax revenue to be slightly greater than 1 in all three groups of countries: 1.10 for developed economies and 1.15 for both emerging economies and low-income countries. However, within each group of countries, they found a wide dispersion of buoyancy estimates obtained for individual countries. Thus, they could not reject a unitary long-run buoyancy for one third of developed countries and half of developing countries. Furthermore, the rejection of unitary buoyancy in the bulk of developed countries appeared to be driven by steep tax increases in the 1970–80s, while estimation on data excluding these two decades produced long-run buoyancy estimates not significantly different from unity in more than half of developed countries.

In their dynamic panel analysis, the MG estimates of the long-run buoyancy of total tax revenues were not statistically significantly different from unity for any group of countries: 0.997 for advanced economies, 1.060 for emerging economies, and 1.210 for low-income countries. Similarly, their PMG estimates were not statistically significantly different from unity in any group of countries: 1.000 for advanced economies, 1.070 for emerging economies, and 1.201 for low-income countries.

Cornevin et al. (2023) used more recent data on 174 countries in 1990–2020 to estimate long- and short-run buoyancy of tax revenues using various estimators. While most estimates of long-run buoyancy of total tax revenues turned out above unity, a unitary long-run buoyancy could not be rejected except for the DCCE-MG estimator in the subsample of low-income countries, where it statistically significantly exceeded unity.

Using a sample of 44 countries in sub-Saharan Africa (SSA) over 1980–2017, Gupta et al. (2022) found long-term tax buoyancy of total tax revenue to be unitary or slightly above unitary in the bulk of those countries. Out of 25 SSA countries that passed the cointegration test, the long-term tax buoyancy of total tax revenue was found to be statistically significantly below unity in three countries, not statistically significantly different from unity in five countries, and statistically significantly above unity in the remaining 17 countries. They also performed dynamic panel analysis on a more balanced panel of 40 SSA countries. The long-term buoyancy of total tax revenue was estimated to be 1.087 using the MG estimator and 1.078 using the PMG estimator.

Appendix A provides a table summarizing the estimates of long-run buoyancy obtained in aforementioned studies. Overall, there appears to be an inverse relationship between the level of economic development and the magnitude of long-run tax buoyancy. In particular, for the

advanced economies, the long-run buoyancy statistically significantly exceeds unity only before 1990 but not thereafter, especially when the sample includes recent additions to OECD from Eastern Europe, which went through the retrenchment of the public sector in the 1990s.

Estimates of the impact of economic development on tax capacity

While the double logarithmic functional form was initially proposed in the studies of the tax-to-GDP ratio as far back as in Williamson (1961), it was not consistently followed thereafter.³ Here I only cite those estimates of the impact of economic development on the tax-to-GDP ratio that were produced using the double logarithmic specification, thus making them comparable to the estimates of tax buoyancy. Generally, such regressions can be described with the following econometric equation:

$$\log(t_{it}/y_{it}) = \alpha_i \cdot \log(y_{it}) + m(X_{it}; \beta_i) + c_i + \eta_i + u_{it} + v_{it} \quad (2)$$

Where

y_{it} stands for GDP in constant USD per capita

t_{it} stands for tax revenues in constant USD per capita

α_i is the impact of economic development

β_i is the impact of other observable factors

$c_i + \eta_i + u_{it} + v_{it}$ captures the impact of all unobservable factors, both transient and time-invariant, common for all countries as well as country-specific.

³ Lotz and Morss (1967) justified their use of a quadratic rather than a logarithmic function by stating that the latter would not be defined for the same values of their explanatory variables, such as trade openness. Bahl (1971) chose not to report his estimates from a double-logarithmic specification as it had lower explanatory power (R-squared) than his preferred specification. Gupta (2007) used logs for economic development but not for the dependent variables, tax-to-GDP ratio.

Williamson (1961) obtained the coefficient of 0.29 in the between regression using a double logarithmic specification without any other covariates for 33 countries, both industrialized and developing, over 1951–57.

In their various stochastic frontier specifications, Canavire-Bacarreza et al. (2021) obtained values around 0.16 for the coefficient on the log of GDP per capita using a double logarithmic specification with other covariates for a balanced panel of 108 countries, both industrialized and developing, over 2002–17.

When using a double logarithmic specification in their fixed effects analysis of 120 countries over the period 1975–98, Teera and Hudson (2004) found the impact of economic development to be statistically insignificant on the sample of all countries but statistically significant for low-middle-income countries (-0.187) and for high-income OECD countries (0.220). When estimating the same specification using a random-effect model, they found the impact of economic development to be statistically insignificant for all groups of countries except low-middle income countries (-0.188).

Using a double logarithmic specification on a sample of 94 countries over the period 1970–2009, Cyan et al. (2014) found the impact of economic development to be around 0.01 in various fixed-effects specifications, while around 0.1 in various specifications of their stochastic frontier analysis.

Mawejje and Sebudde (2019), Fenochietto and Pessino (2013), and Langford and Ohlenburg (2016) allowed for a non-linear impact of economic development on the tax-to-GDP ratio by including a quadratic term of the log of per capita GDP as an additional explanatory variable.

This implies that the impact of economic development on the tax-to-GDP ratio could potentially reverse its sign at some level of economic development. However, for the range of values in the

actual data samples used in Mawejje and Sebudde (2019) and Fenochietto and Pessino (2013), the obtained estimates imply a positive impact of economic development on the tax-to-GDP ratios in all countries. In Langford and Ohlenburg (2016), the coefficient on the log of GDP per capita turned out to be insignificant and was therefore not included in the specification reported in their paper.

Reconciling Estimates between the Two Strands of Literature

The empirical estimates of the impact of economic development on tax outcomes in the short- and long-run, respectively, can be related to the estimates of tax buoyancy reported in the previous studies.

Indeed, by rearranging terms in equation (2), the impact α of economic development on the tax-to-GDP ratio can be related to the elasticity coefficient $\theta = 1 + \alpha$ in the regression of $\log(t_i)$ on $\log(y_i)$:

$$\log(t_i) = (1 + \alpha) \log(y_i) + m(X_{it}; \beta) + c_i + \eta_i + u_{it} + v_{it}$$

Note, however, that in the tax effort studies, tax revenues t_i and the level of economic development y_i are measured in constant USD per capita while tax buoyancy is typically estimated in nominal terms. In Appendix B, I show that that an estimate of long-run buoyancy can be translated into the magnitude of the impact of economic development on the tax-to-GDP ratio by making adjustments for how the population size and real exchange rate interact with economic growth:

$$\theta_i = E_y[t/y] = \{E_Y[T] - 1\} * \{1 + E_y[(N * X)]\}$$

Where

Y stands for nominal GDP

y stands for GDP in constant USD per capita

T stands for nominal tax revenues

t stands for tax revenues in constant USD per capita

N stands for the country's population

X stands for the real exchange rate (local currency units per constant USD)

$E_x[f(x)]$ is the point elasticity operator for function $f(\cdot)$ with respect to argument x , for example $E_Y[T]$ expresses the elasticity of nominal tax revenues (T) with respect to nominal GDP (Y).

For the tax-to-GDP ratio to be increasing with the level of economic development ($\theta_i > 0$), the nominal tax buoyancy has to be more than unitary ($E_Y[T] > 1$), unless the economic development is associated with a considerable depopulation ($E_Y[(N)] < 0$) or currency appreciation ($E_Y[(X)] < 0$) or both.

Empirical Illustration

In this section I use data on 31 advanced economies⁴ from the IMF World Economic Outlook (WEO) dataset (October 2024 release) in order to illustrate the derived correspondence between the long-run buoyancy and the magnitude of the impact of economic development on the tax-to-GDP ratio. The WEO dataset only reports total general government revenues, including both tax and non-tax sources. However, it is still useful for the purpose of illustration.

Table 2. Correspondence between the Long-run Buoyancy and the Magnitude of the Impact of Economic Development

Country	$E_Y[T]$	$E_Y[(N*X)]$	$\{E_Y[T] - 1\} * \{1 + E_Y[(N*X)]\}$	$E_Y[t/y]$
Mean	1.03	2.02	0.07	0.08
Median	1.03	1.73	0.07	0.08
Coefficient of Variation	0.10	0.70	7.03	4.29
Minimum	0.81	0.20	-1.64	-0.52

⁴ Same 31 advanced economies as in Dudine and Jalles (2018).

Q1	0.95	1.25	-0.10	-0.10
Q3	1.07	2.30	0.24	0.20
Maximum	1.28	7.64	1.38	1.32

Table 2 provides a descriptive summary, while Appendix C reports these statistical parameters estimated for each of the 31 countries over 1980–2019. The first column of Table 2 reveals that the values of long-run revenue buoyancy estimated in nominal terms range from 0.81 for Israel to 1.28 in Cyprus, with the average across all countries being 1.03. The next column summarizes the range of values for the statistical parameter that quantifies how the population size and real exchange rate interact with economic growth. This parameter is used in the penultimate column to translate the estimate of long-run buoyancy into the magnitude of the impact of economic development on the revenue-to-GDP ratio. For validation, the last column reports this magnitude of the impact of economic development estimated directly from the data by regressing the revenue-to-GDP ratio on per capita GDP, both measured in constant USD per capita. The sample average (0.07) of the values derived from the buoyancy estimates is quite close to the average (0.08) of the values directly obtained from the regression of the revenue-to-GDP ratio on per capita GDP. The coefficient of correlation between the two columns is 0.9. Appendix C reveals that these values are remarkably close for most of the 31 countries, with a few exceptions comprised of outliers with extreme values of statistical estimates (e.g., 0.81 in Israel). It would require further examination to determine what causes these rare discrepancies, with one possibility being that the double-logarithmic estimate of buoyancy relies on the linearization of the logarithmic function, thus discarding quadratic and high-order terms, which can be non-negligible for observations with extreme values. In any case, this should not be a major concern in the applications focusing on average or “standard” relationships between countries’ characteristics and their tax outcomes and relying on approaches like the Mean Group estimator.

Conclusion and Policy Implications

Conceptually, estimates of long-run tax buoyancy can tell us something about the impact of economic development on tax capacity. However, the relationship between economic development and tax outcomes appears to be nuanced (non-linear, interacting with other factors, etc.).

Empirical evidence from both strands of literature suggests an inverse relationship between the level of economic development and the magnitude of its impact on tax outcomes. However, even when looking at a group of countries belonging to the same class of economic development in a given span of time, estimates of that magnitude still vary depending on the econometric method. This can cast doubt on the credibility of the benchmark synthesized from these estimates and used for evaluating a country's tax effort.

It is my hope that this paper will provide analysts with some guidance on reconciling disparate or conflicting estimates to be used for synthesizing the tax capacity benchmark. In particular, one can refer to the estimates produced with more robust methods in the tax buoyancy literature in order to validate the parameters used to produce the tax capacity benchmark in the tax effort studies. To aid this process, in this paper, I have established correspondence between the statistical parameters estimated in these two strands of literature. Thus, I show that an estimate of long-run buoyancy can be translated into the magnitude of the impact of economic development on the tax-to-GDP ratio by making adjustments for how the population size and real exchange rate interact with economic growth. Among other things, this suggests that the tax effort studies should consider population growth (e.g., Bird et al. 2006) and inflation/depreciation (e.g., Ghura 1998, Langford and Ohlenburg 2016, Mawejje and Sebudde 2019, Canavire-Bacarreza et al. 2021) for inclusion as covariates in tax effort regressions.

In such a validation process, some conflicting estimates could be disregarded if the underlying estimation methodology is not sufficiently robust to the major empirical challenges discussed in this paper. For example, the negative impact of economic development produced with the within estimator can be deemed suspect. Other disparate estimates might be used concurrently to produce a range of estimated tax capacity values to be used for benchmarking the tax effort.

Recent advances in tax buoyancy studies can offer methodological guidance for tax effort studies. In particular, they suggest that tax effort studies should account for common shocks—at a minimum by including time effect dummies in the regression—and test for cross-section dependences (CSD) in the residuals. Furthermore, econometric analysis should allow for some panel heterogeneity, at least for short-run coefficients. Finally, when assessing the tax effort trajectory, one should make adjustments for short-run responses of tax systems to macroeconomic shocks.

References

- Adam, C, D Bevan, and G Chambas. 2001. "Exchange Rate Regimes and Revenue Performance in Sub-Saharan Africa." *Journal of Development Economics* 64 (1): 173–213.
- Bahl, Roy W. 1971 "A regression approach to tax effort and tax ratio analysis." *International Monetary Fund Staff Papers*, XVIII (no. 3), pp. 570-612.
- Belinga, Vincent, Dora Benedek, Ruud A. de Mooij, John Norregaard, 2014. "Tax Buoyancy in OECD Countries," *IMF Working Papers* 2014/110, International Monetary Fund.
- Blundell, R., and Bond, S. (1998). "Initial conditions and moment restrictions in dynamic panel data models." *Journal of Econometrics*, 87(1): 115.
- Bird, Richard M., Jorge Martinez-Vazquez, and Benno Torgler. (2006). "Societal institutions and tax effort in developing countries." *The challenges of tax reform in a global economy.* Springer-Verlag
- Canavire-Bacarreza, Gustavo, Deza, Maria Cecilia, Manzano, Osmel, and Puerta, Alejandro, 2021, "Revisiting tax effort in emerging markets" *Public Finance Review* Vol. 49, No. 6, pp 845, 1552-7530
- Chelliah, Raja J. (1971). "Trends in Taxation in Developing Countries" *International Monetary Fund Staff Papers*, 18, 254–331.
- Chudik, A., and M. H. Pesaran. (2015). "Common correlated effects estimation of heterogeneous dynamic panel data models with weakly exogenous regressors." *Journal of Econometrics*, 188: 393–420.
- Cornevin, A., Corrales, J.S., Angel, J.P. 2023. "A Deep Dive into Tax Buoyancy: Comparing Estimation Techniques in a Large Heterogeneous Panel," *IMF Working Paper No. 23/71*, International Monetary Fund, Washington, D.C.
- Cyan, M., J. Martinez-Vazquez, and V., Violeta. (2014). "New approaches to measuring tax effort." In Richard M. Bird and Jorge Martinez-Vazquez, eds., *Taxation and Development: The Weakest Link (Essays in Honor of Roy Bahl)*, 27–68.
- Deli, Yota & Rodriguez, Abian Garcia & Kostarakos, Ilias & Varthalitis, Petros, 2018. "Dynamic tax revenue buoyancy estimates for a panel of OECD countries," *Papers WP592*, Economic and Social Research Institute (ESRI).
- Dudine, Paolo and Joao Tovar Jalles, 2018. "How Buoyant Is the Tax System? New Evidence from a Large Heterogeneous Panel," *Journal of International Development*, vol. 30(6), pages 961-991.
- Fenochietto, Ricardo, & Carola Pessino. 2013. "Understanding Countries' Tax Effort," *IMF Working Paper No. 2013/244*. Washington, DC, USA: International Monetary Fund.
- Ghura, D. (1998). "Tax Revenue in Sub-Saharan Africa: Effects of Economic Policies and Corruption." *IMF Working Papers*, 1998 (135).
<https://doi.org/10.5089/9781451855685.001>
- Groves, Harold M., and C. Harry Kahn. "The Stability of State and Local Tax Yields." *The American Economic Review*, vol. 42, no. 1, 1952, pp. 87–102.

- Gupta, Abhijit Sen. (2007). *Determinants of Tax Revenue Efforts in Developing Countries* Working paper. No. 7-184. International Monetary Fund.
<https://doi.org/10.5089/9781451867480.001>
- Gupta, S., Jalles, J.T. & Liu, J. (2022). Tax Buoyancy in Sub-Saharan Africa and its Determinants. *Int Tax Public Finance* 29, 890–921 <https://doi.org/10.1007/s10797-021-09694-x>
- Hinrichs, Harley H. 1965, “Determinants of Government Revenue Shares Among Less-Developed Countries.” *The Economic Journal*, vol. 75, no. 299, pp. 546–56. JSTOR, <https://doi.org/10.2307/2228952>.
- Hauk, W.R., Wacziarg, R. A Monte Carlo study of growth regressions. *J Econ Growth* 14, 103–147 (2009). <https://doi.org/10.1007/s10887-009-9040-3>
- Hayo, B., Mierzwa, S. & Ünal, U. Estimating policy-corrected long-term and short-term tax elasticities for the USA, Germany, and the United Kingdom. *Empir Econ* 64, 465–504 (2023). <https://doi.org/10.1007/s00181-022-02252-2>
- Kapetanios G, Pesaran MH, Yamagata T (2011). “Panels with non-stationary multifactor error structures.” *Journal of Econometrics*, 160(2), 326–348.
- Kiviet, Jan; Pleus, Milan; Poldermans, Rutger. (2017). Accuracy and efficiency of various GMM inference techniques in dynamic micro panel data models. *Econometrics*, ISSN 2225–1146, MDPI, Basel, 5(1): 1–54. doi.org/10.3390/econometrics5010014
- Koester, Gerrit and Christoph Priesmeier, 2017. "Revenue elasticities in euro area countries," Working Paper Series 1989, European Central Bank.
- Lagravinese, Raffaele, Paolo Liberati, and Agnese Sacchi. 2020. “Tax buoyancy in OECD countries: New empirical evidence,” *Journal of Macroeconomics*, Elsevier, vol. 63.
- Langford, B., and Ohlenburg, T. (2016). Tax revenue potential and effort. International Growth Centre, Working Paper, S-43202-UGA-1.
- Leuthold, Jane H., 1991, “Tax shares in developing economies: A panel study,” *Journal of Development Economics*, Vol. 35. Pp. 173-85
- Le, Tuan Minh, Blanca-Moreno Dodson and Nihal Bayraktar, 2012, “Tax Capacity and Tax Effort: Extended Cross-Country Analysis 1994-2009,” Policy Research Working Paper 6252, The World Bank.
- Lotz, Jørgen R. and Elliott R. Morss. 1967. “Measuring ‘Tax Effort’ in Developing Countries,” *Staff Papers (International Monetary Fund)*, Vol. 14, No. 3 (Nov. 1967), pp. 478-499.
- Mansfield, Charles Y. “Elasticity and Buoyancy of a Tax System: A Method Applied to,” *Staff Papers (International Monetary Fund)*, vol. 19, no. 2, 1972, pp. 425–46.
- Mawejje, Joseph & Sebudde, Rachel K., 2019. "Tax revenue potential and effort: Worldwide estimates using a new dataset," *Economic Analysis and Policy*, vol. 63(C), pages 119-129.
- Mourre, Gilles and Savina Princen, The Dynamics of Tax Elasticities in the Whole European Union, *CESifo Economic Studies*, Volume 65, Issue 2, June 2019, Pages 204–235, <https://doi.org/10.1093/cesifo/ify028>

- Musgrave, Richard A., and Merton H. Miller. "Built-in Flexibility." *The American Economic Review*, vol. 38, no. 1, 1948, pp. 122–28.
- Newlyn, W.T. (1985), "Measuring Tax Effort in Developing Countries", *Journal of Development Studies*, 21 (3), pp. 390-405.
- Pesaran MH (2006). "Estimation and inference in large heterogeneous panels with a multifactor error structure." *Econometrica*, 74(4), 967–1012.
- Pesaran, M.H. and Smith, R. (1995) Estimating Long-Run Relationship from Dynamic Heterogeneous Panels. *Journal of Econometrics*, 68, 79-113.
- Pesaran, M. Hashem, Yongcheol Shin & Ron P. Smith (1999) Pooled Mean Group Estimation of Dynamic Heterogeneous Panels, *Journal of the American Statistical Association*, 94:446, 621-634, DOI: 10.1080/01621459.1999.10474156
- Sobel, Russell S., and Randall G. Holcombe. 1996. "Measuring the growth and variability of tax bases over the business cycle," *National Tax Journal* 49:535-552.
- Tait, Alan A., Wilfrid L. M. Grätz and Barry J. Eichengreen. 1979. "International Comparisons of Taxation for Selected Developing Countries, 1972-76," *Staff Papers (International Monetary Fund)*, Vol. 26, No. 1, pp. 123-156.
- Teera, J. M. and J. Hudson. (2004). "Tax performance: a comparative study." *Journal of International Development*, 16(6): 785–802.
- Thorn, Richard S. 1967, "The Evolution of Public Finances During Economic Development," *The Manchester School of Economic and Social Studies*, 35: 19-53.
<https://doi.org/10.1111/j.1467-9957.1967.tb01204.x>
- Vito Tanzi, Clayton McCuiston, Determinants of Government Revenue Shares Among Less-Developed Countries: A Comment, *The Economic Journal*, Volume 77, Issue 306, 1 June 1967, Pages 403–405, <https://doi.org/10.2307/2229337>
- Timofeev, Andrey. 2023. "Disentangling Tax Capacity and Effort," 2023, International Center for Public Policy Working Paper 23–21. Andrew Young School of Policy Studies, Georgia State University.
- Weiss, Steven J. "Factors Affecting the Government Revenue Share in Less Developed Countries." *Social and Economic Studies* 18, no. 4 (1969): 348–64
- Williamson, Jeffrey G. (1961) "Public Expenditure and Revenue: An International Comparison," *The Manchester School of Economic and Social Studies*, Vol. XXIX, pp. 43-56.

Appendix A: Summary of Long-run Buoyancy Estimates

Study	Countries	Time span	MG estimate	PMG estimate	DCCE-MG	DCCE-P	Notes
Belinga et al. (2014)	22 OECD countries	1965–2012		1.06			tax revenues inclusive of SSC
		1989–2012		1.02			
		1965–1988		1.09			
Deli et al. (2018)	25 OECD countries	1965–2015	1.03	1.006			total tax revenues inclusive of SSC
Lagravinese et al. (2020)	35 OECD countries	1995–2016				0.91	
Dudine and Jalles (2018)	31 advanced economies (AEs)	1980–2014	0.997	1.000			tax revenues inclusive of SSC
	38 emerging market economies		1.060	1.070			
	38 low-income countries (LIC)		1.210	1.201			
Cornevin et al. (2023)	35 AE	1990–2020	1.076	1.012	1.094	0.979	total tax revenues
	87 EME		1.107	1.080	1.043	1.118	
	52 LIC		1.076	0.983	1.292	1.049	
Gupta et al. (2022)	40 countries in sub-Saharan Africa (SSA)	1980–2017	1.087	1.078			

Notes: Bold font indicates estimates that are statistically significantly different from unity

Appendix B: Tax Capacity and Tax Buoyancy

Let $E_x[f(x)]$ denote the elasticity operator for function $f()$ with respect to argument x . for example $E_Y[T]$ expresses the elasticity of nominal tax revenues (T) with respect to nominal GDP (Y). Further, let us use the following notation:

N stands for the country's population

X stands for the "real" exchange rate (local currency units per constant USD)

Y stands for nominal GDP

y stands for GDP in constant USD per capita, i.e. $y = Y/(N*X)$

T stands for nominal tax revenues

t stands for tax revenues in constant USD per capita, i.e. $t = T/(N*X)$

Let's recall some useful properties of the elasticity operator:

$$E_x[f(x)*g(x)] = E_x[f(x)] + E_x[g(x)] \quad (A.1)$$

$$E_x[f(x)/g(x)] = E_x[f(x)] - E_x[g(x)] \quad (A.2)$$

$$E_x[f(g(x))] = E_g[f(g)] * E_x[g(x)] \quad (A.3)^5$$

Using (A.2), one can derive

$$E_y[t/y] = E_y[t] - E_y[y] = E_y[t] - 1$$

and

$$E_y[t] = E_y[T/(N*X)] = E_y[T] - E_y[(N*X)].$$

Using (A.1), one can derive

$$E_y[Y] = E_y[y*(N*X)] = 1 + E_y[(N*X)].$$

⁵ $E_x[f(g(x))] = x * f'(\cdot) * g'(\cdot) / f(\cdot) = [f'(\cdot) * g'(\cdot) / f(\cdot)] * [x * g'(\cdot) / g(\cdot)] = E_g[f(g)] * E_x[g(x)]$.

Finally, using (A.3), one can derive

$$E_y[T] = E_Y[T] E_y[Y]$$

Combining all the equations above,

$$E_y[t/y] = E_Y[T] E_y[Y] - E_y[(N^*X)] - 1 = E_Y[T] \{1 + E_y[(N^*X)]\} - E_y[(N^*X)] - 1$$

After collecting the terms, we get

$$E_y[t/y] = \{E_Y[T] - 1\} + \{E_Y[T] - 1\} E_y[(N^*X)] = \{E_Y[T] - 1\} * \{1 + E_y[(N^*X)]\}$$

Appendix C: Tax Capacity and Tax Buoyancy

Country	$E_y[T]$	$E_y[X]$	$E_y[N]$	$E_y[N*X]$	$1 + E_y[N*X]$	$\{E_y[T] - 1\}^* \{1 + E_y[N*X]\}$	$E_y[t/y]$
AUS	0.89	0.88	0.23	1.11	2.11	-0.23	-0.22
AUT	1.06	1.25	0.25	1.51	2.51	0.16	0.15
BEL	1.03	1.36	0.24	1.59	2.59	0.09	0.08
CAN	1.00	1.64	0.74	2.38	3.38	0.00	-0.03
CYP	1.28	1.21	0.55	1.76	2.76	0.78	0.77
CHE	1.16	1.23	0.74	1.96	2.96	0.47	0.44
CZE	1.05	0.90	0.06	0.96	1.96	0.10	0.11
DEU	1.02	1.25	0.13	1.38	2.38	0.06	0.03
DNK	1.04	1.53	0.20	1.73	2.73	0.11	0.12
ESP	1.10	2.03	0.31	2.34	3.34	0.32	0.32
EST	1.00	1.35	-0.09	1.26	2.26	0.00	-0.01
FIN	1.03	1.26	0.18	1.45	2.45	0.07	0.03
FRA	1.05	1.36	0.34	1.70	2.70	0.14	0.13
GBR	1.00	1.61	0.22	1.83	2.83	-0.01	0.00
GRC	1.25	4.35	0.25	4.59	5.59	1.38	1.32
IRL	0.85	0.81	0.25	1.05	2.05	-0.31	-0.48
ISL	1.07	3.84	0.54	4.38	5.38	0.37	0.42
ISR	0.81	6.48	1.16	7.64	8.64	-1.64	-0.52
ITA	1.10	3.18	0.10	3.29	4.29	0.41	0.19
JPN	1.14	0.06	0.14	0.20	1.20	0.17	0.21
KOR	1.16	0.73	0.14	0.87	1.87	0.29	0.27
LUX	0.95	0.93	0.41	1.34	2.34	-0.11	-0.22
NLD	0.93	0.96	0.28	1.24	2.24	-0.16	-0.17
NOR	1.07	1.92	0.34	2.26	3.26	0.24	0.26
NZL	0.89	2.06	0.78	2.84	3.84	-0.41	-0.39
PRT	1.07	2.75	0.08	2.83	3.83	0.25	0.19
SGP	0.95	0.39	0.60	0.99	1.99	-0.09	-0.09
SVK	0.93	0.78	0.01	0.79	1.79	-0.13	-0.10
SVN	1.02	1.67	0.06	1.73	2.73	0.05	0.03
SWE	0.89	1.58	0.28	1.86	2.86	-0.31	-0.34
USA	1.00	1.28	0.56	1.84	2.84	0.01	0.08