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Working Paper 26-15

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March 2026

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Estimation and Inference in Quantile Regressions with Multiple Fixed Effects*

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Abstract

This paper proposes a method to estimate quantile regression models with multiple fixed effects. We extend the quantile–via–moments estimator of Machado and Santos Silva (2019) and suggest a computationally efficient Frisch–Waugh–Lovell residualization to partial out additive fixed effects in both the location and scale equations. A unified influence-function inference framework is derived, accommodating heteroskedasticity-robust, clustered, and feasible GLS standard errors. Monte Carlo simulations provide strong support for the validity of the proposed procedure in applications with multi-way unobserved heterogeneity and intra-cluster correlated disturbances. An empirical application to Climate Growth-at-Risk illustrates how temperature shocks affect the conditional distribution of macroeconomic outcomes in a panel of 194 countries. Our findings suggest that in low income countries, downside risks to growth are more strongly linked to temperature shocks than the central tendency or upside risks.

Keywords: Quantile regressions; Location-scale models; Fixed effects; Temperature shocks; Growth-at-Risk

1 Introduction

Quantile regression (QR), introduced by Koenker and Bassett (1978), is a statistical technique that allows researchers to estimate the conditional quantiles of a target variable given a set of predictors. A relatively recent advance in the literature has extended QR to panel data settings with individual fixed effects (FE). These models face an incidental parameters

*The opinions expressed in this paper are those of the authors and do not necessarily reflect the views of the World Bank, Bank of Spain, their Board of Directors, the countries they represent or the Euro-system. The authors would like to thank Enrique Pinzón, Joao Santos-Silva, and Rómulo Chumacero for their helpful comments and suggestions. All remaining errors are our own.

problem (Neyman and Scott 1948; Lancaster 2000), and a variety of solutions have been proposed including penalized FE estimators (Koenker 2004; Lamarche 2010), correlated random effects approximations (Abrevaya and Dahl 2008), and location-shift specifications (Canay 2011) (see Galvao and Kengo (2017) for a review). However, none of these approaches has become the standard practice in applied work, possibly due to restrictive assumptions on how FE enter the conditional quantiles, computational complexities in high-dimensional panels, and implementation frictions.

More recently, Machado and Santos Silva (2019) (MSS) proposed the quantile via moments estimator (MMQREG), assuming a conditional location–scale model similar to He (1997) and Zhao (2000), with a linear in parameters multiplicative heteroskedasticity. In panel data settings, the MMQREG framework accommodates individual FE in both location and scale, allowing the FE to shift the entire conditional distribution, rather than imposing a pure location shift as in Koenker (2004), Lamarche (2010), or Canay (2011). Estimation proceeds by estimating three equations via the Method of Moments (MM): (i) location, (ii) scale, and (iii) local-quantile. MSS derive the large-sample distribution of the estimator and discuss inference based on asymptotic and bootstrap standard errors. Under its identifying assumptions, the MMQREG approach delivers a fast, transparent, and easily implementable alternative to estimate the heterogeneous effects of predictors across the conditional distribution of an outcome.

This paper extends the MMQREG framework in two dimensions. First, we allow for the inclusion of multiple types of FE entering both the location and the scale equations (e.g. country and year or worker and firm) and suggest estimation through a sequence of least-squares problems in which FE are absorbed via Frisch–Waugh–Lovell (FWL) residualization, enabling computation in large multi-way panels without explicit dummy expansion. Second, building on the properties of the Generalized Method of Moments (GMM) estimator, we derive an influence-function representation for the full parameter vector underlying the MMQREG procedure and use it to construct heteroskedasticity-robust and cluster-robust variance estimators, with straightforward extensions to multi-way clustering. Third, under the canonical location–scale model we provide a feasible GLS variance benchmark and document, via Monte Carlo experiments, when it can be unstable in finite samples (e.g., when estimated scales approach zero), in which case IF-based robust/clustered inference is more reliable.

An application of our methodology to Climate Growth at Risk (CGaR) reveals the value of accounting for several dimensions of FE to uncover heterogeneous effects of climate shocks on the conditional distribution of economic outcomes. Specifically, the empirical analysis suggest that climate shocks have significant impacts on GDP growth in poor countries, and the effect is stronger in the left tail of the conditional distribution, implying that climate is an important source of downside risk. The effects on other macroeconomic outcomes like real domestic absorption, welfare relevant total factor productivity and prices are also estimated.

The rest of the paper is structured as follows: Section 2 presents the basic econometric setup of the location-scale model in MSS, and describes our proposal for estimation and inference under the existence of multiple dimensions of FE. Section 3 presents the results of a simulation study to validate the properties of our methodological contribution. Section 4 describes the application of the approach to the CGaR case. Finally, Section 5 concludes.

2 Econometric Framework

2.1 Location-Scale Model for Panel Quantile Regressions

Let y_{it} denote the outcome variable for unit $i = 1, \ldots, N$ at period $t = 1, \ldots, T$, and let x_{it} be a $K \times 1$ vector of observed covariates. For a random variable Y with conditional distribution function $F_{Y|X}(y|x)$, the conditional τ-quantile of Y given X is defined as:

$$Q_Y(\tau|x) = \inf\{y : F_{Y|X}(y|x) \geq \tau\}, \qquad \tau \in (0,1). \tag{1}$$

The objective is to estimate the conditional quantile function $Q_Y(\tau|x)$ assuming the following location-scale specification:

$$y_{it} = x_{it}'\beta + \nu_{it}, \qquad \nu_{it} = x_{it}'\gamma \times \varepsilon_{it}\,, \tag{2}$$

where $Pr(x_{it}'\gamma > 0) = 1$ and ε_{it} is an unobserved random variable, independent of X, with density function $f_\epsilon(.)$ bounded away from zero and normalized to satisfy $\mathbb{E}[\varepsilon_{it}] = 0$ and $\mathbb{E}[|\varepsilon_{it}|] = 1$. The conditional quantile of y_{it} given x_{it} is:

$$Q_{y_{it}}(\tau|x_{it}) = x_{it}'\beta + q_\tau\, x_{it}'\gamma = x_{it}'(\beta + q_\tau \times \gamma), \tag{3}$$

with q_τ denoting the τ-quantile of the standardized error ε_{it}. Observe that under the imposed data-generating-mechanism the entire quantile process is constrained to lie on a 2-dimensional affine subspace spanned by β and γ, implying that all heterogeneity across quantiles is driven by the single scalar function q_τ multiplying a fixed vector γ. While this is a much stronger restriction than in standard linear QR, it has two advantages. First, because the location-scale model can be identified globally, with only a single parameter q_τ requiring local estimation, this estimation approach will be more efficient than the standard QR model (Zhao 2000). Second, under the assumption that $x_{it}'\gamma$ is strictly positive, the model will produce quantile coefficients that do not cross (He 1997).

Estimation proceeds by exploiting the moment conditions implied by (2) via the MM estimator. Although it is possible to identify all coefficients (β, γ, q_τ) simultaneously, we describe and use the implementation approach in MSS, which identifies each set of coefficients separately. For this purpose, let $\widehat{u}_{it} = y_{it} - x_{it}'\widehat{\beta}$ be the residuals from an OLS regression of y_{it} on x_{it}. Then, β and γ can be consistently estimated as:

$$\widehat{\beta} = \arg\min_{\beta} \sum_{i,t} (y_{it} - x_{it}'\beta)^2, \tag{4}$$

$$\widehat{\gamma} = \arg\min_{\gamma} \sum_{i,t} \big(|\widehat{u}_{it}| - x_{it}'\gamma\big)^2. \tag{5}$$

Let $\widehat{\varepsilon}_{it} = \widehat{u}_{it}/(x_{it}'\widehat{\gamma})$ denote the standardized residuals. The empirical τ-quantile of $\widehat{\varepsilon}_{it}$, $\widehat{q}_\tau$, yields the quantile-specific coefficient vector:

$$\widehat{\beta}(\tau) = \widehat{\beta} + \widehat{q}_\tau\, \widehat{\gamma}. \tag{6}$$

Equations (4)–(6) constitute the MMQREG estimator in the pooled cross-sectional case.

Including individual FEs. The approach in MSS allows for unobserved heterogeneity to operate through a single additive component. The conditional location-scale model becomes:

$$y_{it} = a_i + x_{it}'\beta + \nu_{it}, \tag{7}$$
$$\nu_{it} = \big(\eta_i + x_{it}'\gamma\big) \times \varepsilon_{it}, \tag{8}$$

where a_i and η_i are unit-specific effects in the location and scale equations, respectively. This formulation allows the FE to shift not only the conditional mean but also the conditional variability of the outcome. Under (7)–(8), the conditional quantile of y_{it} given (x_{it}, a_i, η_i) becomes:

$$Q_{y_{it}}(\tau|x_{it}, a_i, \eta_i) = a_i + x_{it}'\beta + \big(\eta_i + x_{it}'\gamma\big) \times q_\tau, \qquad \tau \in (0,1), \tag{9}$$

where q_τ is the τ–quantile of the standardized innovation ε_{it}. Equation (9) shows that heterogeneity across units affects both the central tendency and the entire shape of the conditional distribution, while the quantile slope coefficient is $\beta(\tau) = \beta + q_\tau\,\gamma$. Estimation follows the same sequence of conditional mean regressions as in the pooled case. In the first step, one estimates β by ordinary least squares on the location Equation (7), including a full set of unit dummies $\{a_i\}$:

$$\widehat{\beta}, \widehat{a}_i = \arg\min_{\beta, a_i} \sum_{i,t} (y_{it} - a_i - x_{it}'\beta)^2. \tag{10}$$

Let $\widehat{u}_{it} = y_{it} - \widehat{a}_i - x_{it}'\widehat{\beta}$ denote the estimated residuals. The second step estimates the scale parameters γ and η_i by regressing the absolute residuals on x_{it} and the same set of fixed effects:

$$\widehat{\gamma}, \widehat{\eta}_i = \arg\min_{\gamma, \eta_i} \sum_{i,t} \big(|\widehat{u}_{it}| - \eta_i - x_{it}'\gamma\big)^2. \tag{11}$$

Define standardized residuals $\widehat{\varepsilon}_{it} = \widehat{u}_{it}/(x_{it}'\widehat{\gamma} + \widehat{\eta}_i)$ and let $\widehat{q}_\tau$ be the empirical τ-quantile of $\widehat{\varepsilon}_{it}$. The quantile-specific coefficients are then obtained as the combination of the estimated location and scale coefficients, mediated by the empirical quantile $\widehat{q}_\tau$, as in Equation 6.

2.2 Estimation in Panels with Multiple Fixed Effects

The previous framework can be generalized to allow the inclusion of multiple dimensions of FE. This type of analysis is useful when dealing with employer-employee linked data (Abowd et al. 2006), teacher-student linked data (Harris and Sass 2011), or in the most common case, when the researcher controls for both individual and time FE in panel regressions. Without loss of generality, assume there are two sets of unobserved heterogeneity that are constant across observations whenever they belong to common groups. The data-generating process is as follows:

$$y_{it} = a_i + \lambda_t + x_{it}'\beta + \nu_{it}, \tag{12}$$
$$\nu_{it} = \big(\eta_i + \psi_t + x_{it}'\gamma\big) \times \varepsilon_{it}, \tag{13}$$

where (a_i, λ_t) capture additive unit and time FE in the location equation, and (η_i, ψ_t) are the corresponding effects in the scale equation. Observe that the same FE structure is assumed in both equations. If the dimension of the groups is low, the model can be estimated following a dummy inclusion approach. However, if either N or T is large, the direct inclusion of the FE as dummy variables is computationally prohibitive. To address this challenge, we employ a Frisch–Waugh–Lovell (FWL) residualization to *partial out* the impact of group FE on both the outcome and regressor variables in the location and scale equations. In the case of unbalanced setups with multiple FE, the estimation involves iterative demeaning processes like Correia (2016), Gaure (2013), Rios-Avila (2015), among others.

The approach proceeds in the following way. First, for the dependent variable and all regressors in the location model, partial-out the group FE using:

$$y_{it} = a_i^y + \lambda_t^y + u_{it}^y, \tag{14}$$

$$x_{it}^j = a_i^{x^j} + \lambda_t^{x^j} + u_{it}^{x^j}, \tag{15}$$

where x_{it}^j, $j = 1, ...K$, is the j-th element of x_{it}. FWL residualization in Equations (14) and (15) is implemented through iterative demeaning: for any variable, we repeatedly subtract within-group means for each FE dimension in turn until convergence. The resulting transformation satisfies the normal-equation orthogonality conditions with respect to all dummy columns and coincides with the residuals from the corresponding least-squares regression with explicit dummies.

Second, obtain the centered-residualized variables as:

$$y_{it}^{rc} = \mathbb{E}[y_{it}] + \hat{u}_{it}^y, \tag{16}$$

$$x_{it}^{j,rc} = \mathbb{E}[x_{it}^j] + \hat{u}_{it}^{x^j}, \tag{17}$$

and estimate the location equation using the centered-residualized variables in a regression of the form:

$$y_{it}^{rc} = x_{it}^{rc\prime}\beta + \nu_{it}. \tag{18}$$

Because $|\hat{\nu}_{it}|$ is the dependent variable in the scale model, we partial-out and center this term, then estimate the following model:

$$|\hat{\nu}_{it}|^{rc} = x_{it}^{rc\prime}\gamma + w_{it}. \tag{19}$$

Finally, the standardized residuals ϵ_{it} are obtained as follows:

$$\hat{\epsilon}_{it} = \frac{\hat{\nu}_{it}}{|\hat{\nu}_{it}| - \hat{w}_{it}}, \tag{20}$$

where $|\hat{\nu}_{it}| - \hat{w}_{it}$ is the prediction for the conditional standard deviation $\sigma(x_{it}, \eta_i, \psi_t) = \eta_i + \psi_t + x_{it}'\gamma$ and $\hat{w}_{it}$ is the residual from Equation (19). Empirical quantiles $\widehat{q}_\tau$ are computed analogously to the pooled and the cross-sectional case.

Two important clarifications are worth mentioning. First, the theory requires the scale term to be strictly positive. However, in the implementation, the scale equation is estimated as a linear regression with FE without any explicit non-negativity constraint. A possibility is to consider a log-scale specification, e.g. $\sigma(x_{it}, \eta_i, \lambda_t) = exp\big(\eta_i + \psi_t + x_{it}'\gamma\big)$, which enforces positivity by construction and yields a natural multiplicative interpretation of risk. If the linear-scale version is maintained, applications should report the distribution of the fitted scale to document how well the theoretical positivity condition holds in practice. Second, the FWL residualization is equivalent to running the full dummy-variable regressions and does not change change the estimand or its asymptotic distribution. The gain from using iterative demeaning is therefore computational rather than statistical as it eliminates the need to store and invert very large dummy matrices, which is critical in high-dimensional two-way or multi-way FE settings, as in Correia (2016) and Gaure (2013).

2.3 Standard Errors: GLS, Robust, Clustered

The estimation of quantile coefficients under the conditional location-scale model described above can be framed within the Generalized Method of Moments (GMM) framework. For simplicity in notation, we omit the subindex t and assume that any type of fixed effects are already contained in x_i. Let the error term $\nu_i = y_i - x_i'\beta$ and define the following set of orthogonality conditions:

$$\begin{aligned} \mathbb{E}[x_i \nu_i] &= \mathbb{E}[h_{1,i}] = 0, \\ \mathbb{E}\big[x_i\big(|\nu_i| - x_i'\gamma\big)\big] &= \mathbb{E}[h_{2,i}] = 0, \\ \mathbb{E}\left[1\left(q_\tau \geq \frac{y_i - x_i'\beta}{x_i'\gamma}\right) - \tau\right] &= \mathbb{E}[h_{3,i}] = 0. \end{aligned} \tag{21}$$

The first two moment conditions identify the location and scale parameters (β, γ), while the third condition identifies the quantile q_τ of the standardized innovation $\varepsilon_i = \nu_i/(x_i'\gamma)$. Under the regularity conditions discussed in Newey and McFadden (1994, Section 7), Cameron and Trivedi (2005, Ch. 6.3.9), and Machado and Santos Silva (2019), the estimator $\widehat{\theta} = (\widehat{\beta}', \widehat{\gamma}', \widehat{q}_\tau)'$ is asymptotically normal:

$$\sqrt{n}(\widehat{\theta} - \theta_0) \xrightarrow{d} \mathcal{N}(0, V(\theta_0)),$$

where $V(\theta_0)$ is consistently estimated by

$$\widehat{V}(\widehat{\theta}) = \frac{1}{N}\,\bar{G}(\widehat{\theta})^{-1}\left(\frac{1}{N}\sum_i h_i(\widehat{\theta})\,h_i(\widehat{\theta})'\right)\bar{G}(\widehat{\theta})^{-1}, \qquad \bar{G}(\widehat{\theta}) = -\frac{1}{N}\sum_i \frac{\partial h_i(\theta)}{\partial \theta'}\Big|_{\widehat{\theta}}. \tag{22}$$

Expression (22) is the Eicker-White heteroskedasticity-consistent estimator of the covariance matrix in the location–scale system, with $\bar{G}(\hat{\theta})$ denoting the Jacobian matrix of the moment equations evaluated at $\hat{\theta}$.

Because the QR coefficients are affine in θ, by the delta-method, their asymptotic distribution is also normal with mean $\beta(\tau) = \beta + q_\tau\,\gamma$ and asymptotic variance:

$$\widehat{V}\Big(\widehat{\beta}(\tau)\Big) = \Xi_\tau\,\widehat{V}(\widehat{\theta})\,\Xi_\tau', \qquad \Xi_\tau = [I_K,\ \widehat{q}_\tau \times I_K,\ \widehat{\gamma}], \tag{23}$$

with I_K denoting the $K \times K$ identity matrix, where K is number of explanatory variables including the constant. While it is possible to estimate the variance-covariance matrix using simultaneous model estimation for a just identified model, it is more efficient to estimate each set of coefficients separately. Afterward, we propose to estimate the variance-covariance matrix using the empirical Influence Functions (IF) of the estimators (see Jann (2020) for an overview of the application and Hampel et al. (2005) for an in-depth review).

Let $\lambda_i(\theta)$ denote the empirical IF for observation i:

$$\lambda_i(\theta) = \bar{G}(\theta)^{-1} h_i(\theta), \qquad \widehat{V}(\widehat{\theta}) = \frac{1}{N^2} \sum_i \lambda_i(\widehat{\theta}) \lambda_i(\widehat{\theta})'. \tag{24}$$

In the MMQREG estimator case, the IF for each component can be written as:

$$\begin{aligned}
\lambda_i(\beta) =& N(X'X)^{-1} x_i (x_i'\gamma) \varepsilon_i, \\
\lambda_i(\gamma) =& N(X'X)^{-1} x_i (x_i'\gamma)(\tilde{\varepsilon}_i - 1), \\
\lambda_i(q_\tau) =& \frac{\tau - 1(q_\tau \geq \varepsilon_i)}{f_\varepsilon(q_\tau)} - \frac{x_i'\gamma \, \varepsilon_i}{\bar{x}_i{}'\gamma} - q_\tau \frac{x_i'\gamma(\tilde{\varepsilon}_i - 1)}{\bar{x}_i{}'\gamma},
\end{aligned} \tag{25}$$

where $\tilde{\varepsilon}_i = |\varepsilon_i|$ and $f_\varepsilon(\cdot)$ denotes the density of the standardized residual. Full derivations are provided in the Appendix. The different types of standard errors estimation will depend on the assumptions imposed for the estimation of $V(\theta)$ as explained below.

2.3.1 Robust Standard Errors

The simplest and most natural covariance estimator uses Equation (24), yielding heteroskedasticity-robust (Eicker-White) standard errors:

$$\widehat{V}_{robust}(\widehat{\theta}) = \frac{1}{N^2} \sum_i \lambda_i(\widehat{\theta}) \lambda_i(\widehat{\theta})'. \tag{26}$$

Robust inference on $\widehat{\beta}(\tau)$ follows directly from Equation (23). This approach requires no assumptions about within-cluster correlation and remains valid under general forms of conditional heteroskedasticity.

2.3.2 Clustered Standard Errors

In many applications, the error term ε_i may be correlated within clusters. Under this data-structure, GLS standard errors could be severely biased. The standard recommendation has been to report block-bootstrap standard errors, clustered at the individual level. However, because we have access to the IF, it is straightforward to estimate one-way clustered standard errors. Let $g = 1, \ldots, N_G$ index the clusters. Define the cluster-level sums of influence functions $S_g(\widehat{\theta}) = \sum_{i \in g} \lambda_i(\widehat{\theta})$. Then the one-way clustered covariance matrix is:

$$\widehat{V}_{clustered}(\widehat{\theta}) = \frac{1}{N^2} \sum_{g=1}^{N_G} S_g(\widehat{\theta}) S_g(\widehat{\theta})', \tag{27}$$

which generalizes the robust estimator to allow arbitrary correlation within clusters. Extensions to multi-way clustering follow Cameron et al. (2011).

2.3.3 GLS Standard Errors

Finally, the standard errors proposed by Machado and Santos Silva (2019) can be interpreted as a feasible GLS estimator valid under a correctly specified heteroskedasticity model. To estimate GLS standard errors, we consider the variance of the location coefficient estimator:

$$\widehat{V}(\widehat{\beta}) = \frac{1}{N}(X'X)^{-1}\left(\sum_i x_i x_i'(x_i'\gamma\,\varepsilon_i)^2\right)(X'X)^{-1}. \tag{28}$$

If $\text{Var}(\varepsilon_i|x_i) = \sigma_\varepsilon^2$ and the functional form of heteroskedasticity is correctly specified, we can apply the law of iterated expectations to obtain:

$$\widehat{V}(\widehat{\beta}) = \sigma_\varepsilon^2\frac{1}{N}(X'X)^{-1}\left(\sum_i x_i x_i'(x_i'\gamma)^2\right)(X'X)^{-1} = \sigma_\varepsilon^2\frac{1}{N}(X'X)^{-1}\widehat{\Omega}_{\beta\beta}(X'X)^{-1}. \tag{29}$$

This expression corresponds to the GLS estimator accounting for the heteroskedasticity used to identify the QR coefficients. More generally, define the modified IF $\tilde{\lambda}_{i,j}$ and standardized residual components $\psi_{i,j}$ for $j = 1, 2, 3$:

$$\begin{aligned}
\tilde{\lambda}_{i,1} &= \tilde{\lambda}_{i,2} = N(X'X)^{-1}x_i(x_i'\gamma), \\
\tilde{\lambda}_{i,3} &= x_i'\gamma, \\
\psi_{i,1} &= \varepsilon_i, \\
\psi_{i,2} &= \tilde{\varepsilon}_i - 1, \\
\psi_{i,3} &= \frac{1}{x_i'\gamma}\frac{\tau - 1(q_\tau \geq \varepsilon_i)}{f_\varepsilon(q_\tau)} - \frac{\varepsilon_i}{\bar{x}_i{}'\gamma} - q_\tau\frac{(\tilde{\varepsilon}_i - 1)}{\bar{x}_i{}'\gamma}.
\end{aligned} \tag{30}$$

Then, the GLS covariance matrix for (β, γ, q_τ) is:

$$\widehat{V}_{gls}(\widehat{\theta}) = \frac{1}{N^2}\begin{pmatrix} \widehat{\sigma}_{11}\widehat{\Omega}_{11} & \widehat{\sigma}_{12}\widehat{\Omega}_{12} & \widehat{\sigma}_{13}\widehat{\Omega}_{13} \\ \widehat{\sigma}_{12}\widehat{\Omega}_{12} & \widehat{\sigma}_{22}\widehat{\Omega}_{22} & \widehat{\sigma}_{23}\widehat{\Omega}_{23} \\ \widehat{\sigma}_{13}\widehat{\Omega}_{13} & \widehat{\sigma}_{23}\widehat{\Omega}_{23} & \widehat{\sigma}_{33}\widehat{\Omega}_{33} \end{pmatrix}, \tag{31}$$

$$\widehat{\Omega}_{jk} = \frac{1}{N}\sum_i \tilde{\lambda}_{i,j}\tilde{\lambda}_{i,k}', \quad \widehat{\sigma}_{jk} = \frac{1}{N}\sum_i \psi_{i,j}\psi_{i,k}. \tag{32}$$

This GLS estimator coincides with that derived in Theorem 3 of Machado and Santos Silva (2019). However, as illustrated in the simulation study, when the scale model predicts values close to zero or the sample is small, the GLS estimator can become numerically unstable and yield excessively large standard errors. In such cases, robust or clustered standard errors based on the IF representation remain more reliable in finite samples.

2.3.4 Inference in the Multiple Fixed Effects Case

In the presence of multiple dimensions of FE, variance-covariance matrices are obtained following the same IF procedures, but replacing all the elements by their demeaned and re-centered equivalents.

3 Simulation Evidence

To evaluate the finite-sample performance of the proposed estimator, we conduct Monte-Carlo experiments designed to highlight the effect of a two-way FE structure and to compare alternative standard error estimators. Relative to Machado and Santos Silva (2019), our design incorporates two additive unobserved components in the location and scale equations, which allows us to assess both bias and efficiency in the presence of multi-way heterogeneity and intra-cluster correlation. For each observation $i = 1, \ldots, N$, let $g_1(i) \in \{1, \ldots, G\}$ and $g_2(i) \in \{1, \ldots, G\}$ denote group memberships for the two location FE, and let $h_1(i) \in \{1, \ldots, G\}$ and $h_2(i) \in \{1, \ldots, G\}$ denote group memberships for the two scale FE. We set $G = 50$ and assign observations independently and uniformly across groups for each dimension.

Data are simulated according to:

$$y_i = \alpha_{1,g_1(i)} + \alpha_{2,g_2(i)} + x_i + \left(2 + x_i + \eta_{1,h_1(i)} + \eta_{2,h_2(i)}\right) \varepsilon_i, \tag{33}$$

where $\alpha_{1,g} \sim \chi^2(1)$, $\alpha_{2,g} \sim \chi^2(1)$, $\eta_{1,g} \sim \chi^2(1)$, and $\eta_{2,g} \sim \chi^2(1)$ are independent across g and mutually independent across components. The regressor is generated as

$$x_i = 0.5\Big(\chi_i + 0.5\big(\alpha_{1,g_1(i)} + \alpha_{2,g_2(i)}\big)\Big), \qquad \chi_i \sim \chi^2(1),$$

so that x_i is correlated with the location FE by construction. The innovation ε_i follows a centered $\chi^2(5)$ distribution, $\varepsilon_i = r_i/5 - 1$ with $r_i \sim \chi^2(5)$, inducing positive skewness and heteroskedasticity consistent with the assumptions of the MMQREG framework.

Additionally, we consider a design in which the innovation exhibits strong intra-cluster correlation. In particular, we replace the i.i.d. innovation in Equation (33) by a cluster-dependent shock κ_i constructed to preserve the location–scale structure while inducing dependence within clusters:

$$y_i = \alpha_{1,g_1(i)} + \alpha_{2,g_2(i)} + x_i + \left(2 + x_i + \eta_{1,h_1(i)} + \eta_{2,h_2(i)}\right) \kappa_i. \tag{34}$$

The regressor x_i and the FE are generated as in the baseline design. To generate κ_i, let $c(i) \in \{1, \ldots, C\}$ denote an additional cluster assignment that is independent of the FE, with $C = 100$ mutually exclusive clusters. We set

$$\kappa_i = \text{inv}-\chi^2_5(r_i)/5 - 1, \qquad r_i = \Phi\Big(\sqrt{0.25}\, s_i + \sqrt{0.75}\, s_{c(i)}\Big),$$

where $s_i \sim \mathcal{N}(0,1)$ varies across observations and $s_{c(i)} \sim \mathcal{N}(0,1)$ is common to all observations within cluster $c(i)$. Here, $\Phi(\cdot)$ denotes the standard normal cumulative distribution function and $\text{inv}-\chi^2_5(\cdot)$ is the inverse cdf (quantile function) of a $\chi^2(5)$ distribution. This construction induces strong intra-cluster correlation in κ_i (and hence in the composite error term) while preserving the conditional heteroskedastic structure implied by the MMQREG location–scale framework.[1] We consider total sample sizes $N \in \{500, 1000, 2000, 4000\}$.

[1] We do not consider misspecification of the scale equation, since this would affect not only inference but also the bias of the coefficient estimates.

Table 1: Bias, Simulated Standard error, and Mean Squared Error

	N = 500			N = 1000		
	Mean Bias	SE	MSE	Mean Bias	SE	MSE
τ = **0.25**						
mmqreg	0.169	0.267	0.099	0.092	0.172	0.038
jkc	0.048	0.318	0.104	0.014	0.189	0.036
τ = **0.75**						
mmqreg	-0.050	0.446	0.202	-0.010	0.310	0.096
jkc	0.048	0.546	0.301	0.018	0.339	0.115
	N = 2000			N = 4000		
	Mean Bias	SE	MSE	Mean Bias	SE	MSE
τ = **0.25**						
mmqreg	0.050	0.119	0.017	0.026	0.084	0.008
jkc	0.006	0.126	0.016	0.003	0.087	0.008
τ = **0.75**						
mmqreg	0.001	0.215	0.046	0.003	0.151	0.023
jkc	0.006	0.222	0.049	0.002	0.154	0.024

Note: mmqreg - The proposed estimator. JKC-Jacknife Bias Corrected Estimator. SE- Simulated Standard Error. MSE - Mean Squared Error. Mean bias is the difference between the estimated coefficient and the analytical true value.

Given $C = 100$ clusters, these values correspond to an average cluster size of approximately 5, 10, 20, and 40 observations, respectively.

The estimator is implemented using our extension of the MMQREG framework to the multiple FE case, with location and scale equations estimated sequentially via least-squares on the centered residualized variables, as described in Section 2. Table 1 reports the bias, simulated standard error, and mean squared error of the QR coefficient, using the first data generation structure and for 25th and 75th conditional quantiles. We report the baseline multiple FE estimator (denoted *mmqreg*) and a bias-corrected version based on the split-panel jackknife correction of Dhaene and Jochmans (2015) (denoted *jkc*). Similar to the findings in Machado and Santos Silva (2019), the proposed estimator exhibits small but noticeable bias when the sample size is small ($N = 500$), which declines as the number of observations per group increases. The bias seems to be proportional to the sample size, or more precisely, to the average number of observations per sub-group. Interestingly, the jackknife correction nearly eliminates this bias at the 25th quantile, even for small N, though at the cost of increased dispersion. In contrast, when considering the 75th percentile, the `mmqreg` estimator shows smaller bias than the `jkc` estimator. In either case, despite the bias reduction obtained using the `jkc` alternative, the standard errors are larger than without correction. For the 25th quantile, the reduction in bias is large enough to produce a smaller MSE than the `mmqreg` estimator.

To evaluate the performance of the different types of standard errors under within- cluster dependence, we compute the bias, simulated standard errors, average and median standard

Table 2: 95% Coverage Ratio, and Standard Error Estimation: No Intra-cluster Correlation

	τ = **0.25**				τ = **0.75**			
N	*500*	*1000*	*2000*	*4000*	*500*	*1000*	*2000*	*4000*
Bias	0.173	0.096	0.050	0.026	-0.040	-0.008	0.003	0.003
Sim SE	0.268	0.171	0.118	0.084	0.443	0.314	0.219	0.153
GLS								
Mean SE	1.9e7	0.830	0.401	0.084	7.3e7	1.876	4.259	0.156
Median SE	0.495	0.215	0.123	0.083	1.037	0.429	0.225	0.152
CR95%	0.988	0.980	0.958	0.948	0.991	0.977	0.967	0.952
Robust								
Mean SE	0.224	0.159	0.112	0.080	0.353	0.269	0.199	0.144
CR95%	0.892	0.928	0.939	0.932	0.875	0.904	0.927	0.936
Clustered								
Mean SE	0.235	0.162	0.112	0.079	0.372	0.274	0.199	0.143
CR95%	0.907	0.930	0.932	0.930	0.892	0.905	0.925	0.932

errors, and 95% coverage of the biased corrected estimates, for the GLS, robust, and clustered standard errors. The bias magnitudes in Tables 2 and 3 are comparable to the values in Table 1, with simulated standard errors that are slightly larger when we allow for intra-cluster correlations. Under the DGP without intra-cluster correlation (Table 2), the coverage associated to GLS standard errors is above 95% when the samples are small, but it approximates to 95% as the sample size increases. The coverage rates associated to robust standard errors are closer to 90%, albeit increasing slightly for larger samples. One of the main reasons why the GLS standard errors achieve higher than expected rates of coverage may be related to the fact that the standard error estimator is very sensitive to near zero predictions from the scale model. As shown in Table 2, average and median GLSE standard errors are considerably larger than the simulated standard errors when the sample sizes are small. While robust standard errors are less sensitive to this problem, producing more stable results, they tend to underestimate the magnitude of the true standard errors in small samples. This translates into the lower coverage rates. When intra-cluster correlation is introduced, we still observe similar problems with the GLS standard errors, albeit with high coverage rates (see Table 3). While robust and clustered standard errors are smaller in average than the simulated standard errors, coverage rates are above 90%. Clustered standard errors perform the best only when the sample size is large, with robust standard error producing low standard errors estimates.

Our simulation results confirm two main conclusions. First, the proposed multiway FE MMQREG estimator performs well in finite samples, with bias that is small, predictable, and largely mitigated by the jackknife correction. Second, the extended influence-function-based inference procedures yield accurate standard errors, with clustered versions providing the most robust performance under correlated errors and large sample size. In applications, we recommend robust or clustered standard errors to be the default, while GLS standard errors are only advisable when the scale is well behaved and samples are large. Together, these findings provide strong support for the use of the proposed estimator in empirical applications

Table 3: 95% Coverage Ratio, and Standard Error Estimation: With Intra-cluster Correlation

	τ = **0.25**				τ = **0.75**			
N	*500*	*1000*	*2000*	*4000*	*500*	*1000*	*2000*	*4000*
Bias	0.180	0.091	0.053	0.027	-0.022	-0.008	0.003	0.003
Sim SE	0.299	0.200	0.141	0.100	0.503	0.352	0.253	0.182
GLS								
Mean SE	1.5e7	1.053	0.190	0.093	6.0e7	1.332	0.309	0.167
Median SE	0.476	0.215	0.133	0.092	0.913	0.384	0.236	0.165
CR95%	0.984	0.963	0.939	0.928	0.986	0.963	0.940	0.925
Robust								
Mean SE	0.253	0.179	0.126	0.089	0.402	0.303	0.222	0.160
CR95%	0.896	0.916	0.917	0.915	0.880	0.905	0.917	0.906
Clustered								
Mean SE	0.252	0.180	0.130	0.096	0.397	0.304	0.228	0.172
CR95	0.892	0.915	0.923	0.935	0.875	0.902	0.919	0.928

with multi-way unobserved heterogeneity and potentially correlated disturbances.

4 Application: Climate Growth at Risk

4.1 Background

Following the influential contribution of Dell et al. (2012), an extensive amount of literature in climate economics have estimated the effects of temperature changes or shocks on economic growth and other macroeconomic outcomes (Burke et al. 2015; Acevedo et al. 2020; Kahn et al. 2021; Newell et al. 2021; Nath et al. 2024; Kotz et al. 2024; Bearpark et al. 2025; Mohaddes and Raissi 2025; Neal et al. 2025). Panel-data studies generally find that rising temperatures reduce output growth, with damages varying by countries' economic characteristics. For example, Dell et al. (2012) find that a 1-degree Celsius increase in country-level average temperature has significant negative growth effects in poor countries. Burke et al. (2015) allow for nonlinearities and find an inverted-U relationship, with growth peaking around 13.1 degrees. Responses in rich and poor countries are not statistically different. More recently, Neal et al. (2025) revisit panel approaches by adding the global average temperature as regressor. This specification implies substantially larger projected macroeconomic damages compared to models featuring country-level temperatures only.

Most studies quantify climate effects on the conditional mean of output. A few recent exceptions—Kiley (2024), Giraldo et al. (2025), and Chauvet et al. (2025)—apply the Growth-at-Risk (GaR) framework (Adrian et al. 2019; Plagborg-Møller et al. 2020; Brownlees and Souza 2021; Carriero et al. 2025) to analyze shifts on the entire conditional growth distribution using panel-quantile regressions. Kiley (2024) follows the quantile via moments method of Machado and Santos Silva (2019) to obtain that the impact of temperature on the 10th percentile of the growth distribution is about 50% larger than the effect on the median. For Europe, Chauvet et al. (2025) show that worsening extreme weather conditions depress the whole growth distribution, with much stronger effects on downside risk than on upside potential. Giraldo et al. (2025) implement the Abrevaya and Dahl (2008) approach and obtain similar results for Latin America and the Caribbean. The latter reference looks not only at changes in climate averages but also in volatility and higher-order moments.

Despite the use of fixed effects in these studies, there is no systematic discussion of the incidental parameters problem in panel quantile regressions or of estimation with multiple fixed effects. For example, Giraldo et al. (2025) include only country fixed effects and assume they can be approximated through a linear projections on observables, without specifying the projection's functional form. Chauvet et al. (2025) also assume country fixed effects and allow for linear and quadratic time trends. Their panel-quantile specification treats country effects as quantile-invariant, but the estimation approach is not clearly described. Kiley (2024) considers both country and year fixed effects and estimation follows the standard Machado and Santos Silva (2019) two-step strategy, i.e., introducing both dimensions of fixed effects in the conditional-mean stage while leaving the second step unchanged. Our application is closest to Kiley (2024), but we instead implement our proposed quantile-regression estimator with multiple fixed effects based on a FWL decomposition.

4.2 Empirical Strategy

Let Y_{it} denote the output growth rate of country i in period t, and let C_{it} be the climate variable or shock of interest. Our empirical strategy focuses on modeling the conditional distribution of Y_{it} as a function of C_{it}, controls X_{it}, and FE. We adopt the location–scale specification:

$$\begin{aligned} Y_{it} &= \beta_1 C_{it} + X_{it}'\beta + a_i + \lambda_t + v_{it}, \\ v_{it} &= (\gamma_1 C_{it} + X_{it}'\gamma + \eta_i + \psi_t) \times \epsilon_{it}. \end{aligned} \tag{35}$$

Observe that under this QR specification, C_{it} affects the conditional distribution of Y_{it} contemporaneously at time t: it implicitly implies that the climate shock translates into a permanent shift in the level of output from t onwards, as the model does not include a mechanism for subsequent rebound or convergence. If we think of the climate shock as capturing persistent changes in climatic conditions, the interpretation of our below results can be interpreted as reduced-form proxies for longer-run damage.

Temperature shocks. There is no consensus regarding the most convenient temperature transformation. Some studies consider temperature levels directly (e.g., Dell et al., 2012; Burke et al., 2015; Nath et al., 2024; Kiley, 2024), while others defend the use of shocks (e.g., Bilal and Känzig 2024; Nath et al. 2024). Because output growth is typically stationary whereas temperature is upward trending (see Gadea and Gonzalo 2020; Gadea et al. 2024), our preferred specification uses temperature shocks that remove the trending component in the regressor.

Shocks are constructed as deviations of each country's annual average temperature from a time-varying historical norm (Vose et al. 2014; Mohaddes and Raissi 2025). Let T_{it} denote annual average temperature in country i and year t. The rolling historical mean over the preceding $m = 30$ years is obtained as

$$T^*_{it-1}(m) \;=\; \frac{1}{m}\sum_{s=t-m}^{t-1} T_{is}, \tag{36}$$

and the temperature shock as

$$C_{it} \;=\; T_{it} - T^*_{it-1}(m). \tag{37}$$

Because these shocks are auto-correlated, we remove autocorrelation using an $AR(2)$ model and consider the residuals. The robustness or our findings to alternative choices of C_{it} is assessed.

Fixed effects and controls. The empirical specification includes a rich set of FE to account for unobserved heterogeneity across both the cross-sectional and temporal dimensions. Country FE absorb time-invariant characteristics such as geography, institutional quality, and long-run development levels that could jointly influence both macroeconomic performance and exposure to climate conditions. Region×Year fixed effects capture common shocks and global or regional factors that simultaneously affect all units, including worldwide technological progress, synchronized policy responses, or large-scale changes in climate patterns (Neal et al. 2025). This saturated design ensures that identification is based on within-country deviations from long-run averages rather than on persistent cross-sectional differences. In some specifications, we additionally control for standard financial indicators with predictive content for macroeconomic outcomes, as traditionally employed in the Growth-at-Risk literature (Furceri et al. 2025), to absorb cyclical financial conditions unrelated to temperature dynamics.

Identification. Identification of the causal effect of temperature on the conditional distribution of output growth relies on the quasi-exogeneity of the constructed temperature shocks. These shocks represent innovations in the climate process that are orthogonal to predictable variation and are plausibly exogenous to contemporaneous economic decisions (Bilal and Känzig 2024; Nath et al. 2024). The inclusion of both unit and time (or Region×Year) FE further purges the identifying variation from confounding due to latent factors or global trends in output and climate. Consequently, identification stems from the combination of (i) the FE structure, which eliminates persistent cross-sectional heterogeneity, and (ii) the quasi-random timing of temperature innovations, which isolates unexpected fluctuations in climate conditions.

4.3 Data and Implementation

Temperature data. Country-level annual temperature series are obtained from the Weighted Climate Dataset Dashboard provided by Gortan et al. (2024). The dashboard processes multiple climate sources and provides spatially aggregated data with global coverage at the country or regional level. Climate variables are supplied with alternative economic weights (e.g., population density, night-time lights, cropland). In our analysis, we use average temperature from the Climatic Research Unit, weighted by 2015 population, spanning 1901–2022.

Economic data. The main dependent variable is real GDP per capita growth (constant 2015 USD), obtained from the World Bank's World Development Indicators (WDI) databse.

From the same source we collect the consumer price index (CPI) (2010=100), as well as country classifications for economic region and income group. Data on real domestic absorption (constant 2021 USD) and welfare relevant total factor productivity levels (USA = 1) are obtained from the Penn World Table, version 11.0 (Feenstra et al. 2015). Financial Stress Index (FSI), Reported Social Unrest Index (RSUI), and World Uncertainty Index (WUI) indicators are taken from the replication package of Furceri et al. (2025).

For estimation, we restrict the analysis to countries with at least 20 years of data on the dependent variable, assembling in the best scenario an unbalanced panel of 189 countries over 1960–2019.

Implementation. The empirical analysis is conducted in `Stata` using the commands `qregfe` and `qregplot` introduced in Rios-Avila et al. (2025).

4.4 Results

Baseline results for GDP growth. Table 4 reports baseline estimates for the conditional mean and conditional quantile regressions of real GDP per-capita growth. Model (1) includes only country fixed effects. Model (2) adds both country and region × year fixed effects. Model (3) augments Model (2) with an interaction between the country-level temperature shock and the global temperature shock, following Neal et al. (2025). Model (4) further controls for the FSI, RSUI, and WUI indicators, commonly used in the GaR literature. Standard errors are clustered at the region × year level. Results are shown for the full sample and separately for poor and non-poor subsamples, where the poor definition follows the World Bank's income classification. Given geography, structural differences, and potential feedback from output to emissions and land use, our estimates across the conditional output distribution should be interpreted as reduced-form relationships under strong exogeneity assumptions, rather than fully identified structural effects.

We begin by examining the conditional mean (Panel A) and the conditional median (Panel D) regressions, which are informative about the effects of temperature shocks on the central part of the GDP growth distribution. Our estimates are aligned with prior work (e.g., Dell et al. (2012); Kiley (2024)): least squares and median regression deliver similar temperature effects, with the adverse impact concentrated in poor countries. In those economies, the conditional mean of growth declines by approximately 1.3–1.8% in response to a 1 degree increase in country-level average temperature, depending on the specification. Crucially, identification of this effect requires both dimensions of fixed effects: omitting region × year fixed effects renders the coefficient statistically insignificant.

Temperature shocks exhibit pronounced heterogeneity across the conditional GDP growth distribution. For the poor subsample, coefficients are negative and significant in the left tail, indicating downside risk to growth from temperature variations. Magnitudes decline monotonically toward the upper tail. For instance, in Model (2) the effect at the 25th conditional quantile is a −2.93 pp decrease in growth per 1 degree increase in country-level temperature, whereas at the 75th quantile the effect is −1.30 pp, and the 90th-quantile coefficient is statistically indistinguishable from zero. These effects are slightly larger in absolute value than those reported by Kiley (2024). The qualitative pattern is robust across specifications, though adding financial controls (FSI, RSUI, WUI) attenuates the estimates, suggesting that part of the transmission operates through financial conditions commonly emphasized in the GaR literature. For the non-poor subsample, we find the opposite sign at the lower tail: positive and significant effects at the 5th and 25th quantiles, together with small, mildly negative coefficients in the upper tail.

Table 4: Conditional mean and quantile regressions for GDP growth (Baseline)

	Full Sample				**Poor**				**Non-poor**			
Model	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
					Panel A: Conditional mean							
C_{it}	-0.394	0.002	-0.062	-0.293	-0.726*	-1.892***	-2.054***	-1.571**	-0.296	0.401	0.318	0.138
	(0.328)	(0.315)	(0.336)	(0.274)	(0.422)	(0.473)	(0.482)	(0.664)	(0.354)	(0.330)	(0.342)	(0.291)
$C_{it} \times C_{gt}$			0.059				-2.141				-0.087	
			(1.922)				(2.450)				(1.882)	
					Panel B: Quantile τ=.05							
C_{it}	1.283*	0.847	0.830	0.115	0.064	-2.639***	-2.483**	-2.553**	1.682**	1.719**	1.590**	1.038*
	(0.680)	(0.620)	(0.637)	(0.524)	(0.861)	(0.920)	(1.006)	(1.065)	(0.750)	(0.683)	(0.707)	(0.555)
$C_{it} \times C_{gt}$			-1.579				-3.323				-2.190	
			(3.783)				(5.570)				(4.045)	
					Panel C: Quantile τ=.25							
C_{it}	0.108	0.289	0.244	-0.144	-0.506	-2.157***	-2.210***	-1.957***	0.305	0.862**	0.751*	0.484
	(0.402)	(0.398)	(0.419)	(0.326)	(0.505)	(0.567)	(0.606)	(0.722)	(0.435)	(0.429)	(0.442)	(0.349)
$C_{it} \times C_{gt}$			-0.503				-2.571				-0.802	
			(2.462)				(3.308)				(2.501)	
					Panel D: Quantile τ=.5							
C_{it}	-0.419	-0.005	-0.072	-0.293	-0.755*	-1.882***	-2.046***	-1.556**	-0.302	0.403	0.305	0.147
	(0.315)	(0.309)	(0.328)	(0.269)	(0.410)	(0.465)	(0.472)	(0.650)	(0.339)	(0.324)	(0.331)	(0.281)
$C_{it} \times C_{gt}$			0.077				-2.118				-0.066	
			(1.883)				(2.389)				(1.843)	
					Panel E: Quantile τ=.75							
C_{it}	-0.950***	-0.296	-0.375	-0.427	-0.982**	-1.620***	-1.892***	-1.195	-0.944***	-0.061	-0.116	-0.144
	(0.280)	(0.270)	(0.283)	(0.285)	(0.392)	(0.501)	(0.505)	(0.746)	(0.302)	(0.278)	(0.280)	(0.298)
$C_{it} \times C_{gt}$			0.635				-1.695				0.631	
			(1.545)				(2.335)				(1.507)	
					Panel F: Quantile τ=.95							
C_{it}	-1.940***	-0.783**	-0.898***	-0.655	-1.425**	-1.175	-1.635**	-0.623	-2.122***	-0.795**	-0.820**	-0.585
	(0.387)	(0.350)	(0.343)	(0.425)	(0.556)	(0.775)	(0.816)	(1.102)	(0.432)	(0.372)	(0.363)	(0.436)
$C_{it} \times C_{gt}$			1.594				-0.988				1.795	
			(1.820)				(3.878)				(1.965)	
Observations	9761	9761	8761	3081	3665	3665	3283	1338	6096	6096	5478	1743
Units	194	194	194	92	71	71	71	40	123	123	123	52
Periods	59	59	50	34	59	59	50	34	59	59	50	34
Country FE (1 = Yes)	1	1	1	1	1	1	1	1	1	1	1	1
Region×Year FE (1 = Yes)	0	1	1	1	0	1	1	1	0	1	1	1

Notes: The table reports estimated coefficients from panel quantile regressions with GDP growth as the dependent variable. C_{it} denotes the country-level temperature shock, constructed as the residuals from an AR(2) model of temperature deviations relative to the preceding 30-year moving average. C_{gt} is defined analogously using global average temperature. Specification (1) includes only country fixed effects; specification (2) adds both country and Region×Year fixed effects; specification (3) augments (2) with an interaction term between C_{it} and C_{gt}; and specification (4) further controls for the Financial Stress Index (FSI), the Risk Sentiment Uncertainty Index (RSUI), and the World Uncertainty Index (WUI). Standard errors are clustered at the Region×Year level. Significance levels are denoted by: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$.

4.5 Robustness and Additional Analyses

Alternative Shocks. Two alternative measures of temperature shocks C_{it}, are considered. First, following Bilal and Känzig (2024), we construct temperature shocks as the two-step-ahead forecast errors from a Hamilton (2018) filter. For each country i, the temperature series T_{it} is projected h periods ahead on its own p lags:

$$T_{i,t+h} = \alpha_i + \sum_{j=1}^{p} \beta_{ij} T_{i,t-j+1} + u_{i,t+h}, \tag{38}$$

where $\widehat{u}_{i,t+h}$ denotes the Hamilton-filtered innovation. The residual

$$C_{i,t+h} = T_{i,t+h} - \widehat{T}_{i,t+h|t} = \widehat{u}_{i,t+h} \tag{39}$$

is interpreted as the unexpected temperature shock. We set $(h, p) = (2, 2)$. Second, we follow Dell et al. (2012) and Kiley (2024) and use the raw temperature levels as the shock variable. Tables 5 and 6 report the results for both specifications. The main findings are qualitatively preserved: in poor countries, downside risks to growth are more strongly linked to temperature than the central tendency or upside risks. The estimated magnitudes are somewhat larger when using temperature levels, suggesting that persistent deviations in average temperature exert slightly stronger effects than purely transitory shocks.

Other macroeconomic outcomes. We next analyze the responses of the conditional distribution of other macroeconomic aggregates. Table 7 reports the estimated coefficients for the growth rate of per-capita real domestic absorption—defined as the sum of real private consumption and investment. Among poor countries, the conditional mean effect under specifications (2) and (3) is approximately -2% and statistically significant, indicating that temperature shocks reduce aggregate demand in a manner comparable to the reduction in output. However, the shape of the conditional distribution differs markedly from that observed for GDP growth. While GDP growth exhibited stronger declines in the lower quantiles—indicating higher downside risks—, domestic absorption reacts more sharply in the upper part of the distribution, with increasingly negative coefficients above the median. For non-poor countries, the relationship is reversed. The conditional responses of domestic absorption are positive and stronger in the lower quantiles, closely mirroring the distributional pattern obtained for GDP growth. The degree of alignment between output and absorption responses reflects the structural role of demand-side adjustment in shaping the macroeconomic resilience to climate shocks.

Table 8 examines the responses of a key supply-side determinant of output, namely the growth rate of welfare-relevant total factor productivity (TFP). This variable captures changes in the efficiency with which economies transform inputs into output, and is therefore directly linked to the long-run supply potential rather than short-term demand fluctuations. For poor countries, temperature shocks are associated with a decline in the conditional mean of TFP growth of approximately -2.4% under specification (2) and -4.4% under specification (4), both statistically significant. Although the estimated coefficients in the quantile regressions are imprecisely estimated, the point estimates exhibit a clear pattern: larger negative effects in the lower quantiles that reduce in magnitude toward the upper tail of the conditional distribution. This monotonic pattern suggests that temperature shocks depress TFP growth during low-productivity states, reinforcing the left-tail risks in output documented earlier. For non-poor countries, the estimated effects are smaller in absolute value and generally statistically insignificant across the conditional distribution.

Finally, we look at the responses of CPI inflation. For broader applications of Inflation-at-Risk using financial indicators as predictors, see Korobilis et al. (2021) and Lopez-Salido and Loria (2024). According to the results in Table 9, the effects on the conditional average of

Table 5: Conditional mean and quantile regressions for GDP growth (Hamilton-filtered shocks)

	Full Sample				Poor				Non-poor			
Model	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
					Panel A: Conditional mean							
C_{it}	-0.165	0.009	-0.005	-0.144	0.008	-1.125**	-1.195***	-0.671	-0.212	0.306	0.295	0.063
	(0.342)	(0.263)	(0.258)	(0.263)	(0.396)	(0.448)	(0.437)	(0.730)	(0.381)	(0.289)	(0.282)	(0.274)
$C_{it} \times C_{gt}$			-1.584				-5.187**				-1.241	
			(1.310)				(2.073)				(1.376)	
					Panel B: Quantile τ=.05							
C_{it}	1.783**	0.734	0.675	0.244	0.991	-2.155***	-2.165***	-2.121*	2.051**	1.495**	1.415**	0.951**
	(0.753)	(0.536)	(0.514)	(0.501)	(0.785)	(0.720)	(0.733)	(1.136)	(0.856)	(0.616)	(0.582)	(0.477)
$C_{it} \times C_{gt}$			-4.998**				-6.279				-5.526**	
			(2.494)				(4.031)				(2.732)	
					Panel C: Quantile τ=.25							
C_{it}	0.405	0.257	0.227	-0.003	0.291	-1.480***	-1.529***	-1.235	0.452	0.721*	0.679*	0.403
	(0.436)	(0.334)	(0.324)	(0.310)	(0.462)	(0.479)	(0.473)	(0.764)	(0.486)	(0.380)	(0.361)	(0.310)
$C_{it} \times C_{gt}$			-2.748*				-5.563**				-2.709	
			(1.624)				(2.391)				(1.741)	
					Panel D: Quantile τ=.5							
C_{it}	-0.193	0.003	-0.009	-0.145	-0.024	-1.119**	-1.184***	-0.648	-0.219	0.307	0.295	0.073
	(0.326)	(0.257)	(0.254)	(0.258)	(0.387)	(0.442)	(0.431)	(0.714)	(0.363)	(0.284)	(0.276)	(0.265)
$C_{it} \times C_{gt}$			-1.563				-5.175**				-1.239	
			(1.288)				(2.048)				(1.353)	
					Panel E: Quantile τ=.75							
C_{it}	-0.799***	-0.246	-0.244	-0.269	-0.312	-0.749	-0.843*	-0.109	-0.941***	-0.108	-0.096	-0.214
	(0.260)	(0.234)	(0.234)	(0.276)	(0.391)	(0.502)	(0.492)	(0.855)	(0.284)	(0.246)	(0.246)	(0.288)
$C_{it} \times C_{gt}$			-0.382				-4.791**				0.258	
			(1.145)				(2.363)				(1.185)	
					Panel F: Quantile τ=.95							
C_{it}	-1.913***	-0.659**	-0.634*	-0.492	-0.878	-0.155	-0.294	0.796	-2.239***	-0.760**	-0.721**	-0.640
	(0.338)	(0.328)	(0.325)	(0.418)	(0.578)	(0.724)	(0.720)	(1.317)	(0.377)	(0.342)	(0.340)	(0.397)
$C_{it} \times C_{gt}$			1.576				-4.173				2.651*	
			(1.451)				(3.721)				(1.523)	
Observations	9761	9761	9761	3081	3665	3665	3665	1338	6096	6096	6096	1743
Units	194	194	194	92	71	71	71	40	123	123	123	52
Periods	59	59	59	34	59	59	59	34	59	59	59	34
Country FE (1 = Yes)	1	1	1	1	1	1	1	1	1	1	1	1
Region×Year FE (1 = Yes)	0	1	1	1	0	1	1	1	0	1	1	1

Notes: The table reports estimated coefficients from panel quantile regressions with GDP growth as the dependent variable. C_{it} denotes the country-level temperature shock, constructed as the innovations in a Hamilton-filter regression with $p = 2$ and $h = 2$. C_{gt} is defined analogously using global average temperature. Specification (1) includes only country fixed effects; specification (2) adds both country and Region×Year fixed effects; specification (3) augments (2) with an interaction term between C_{it} and C_{gt}; and specification (4) further controls for the Financial Stress Index (FSI), the Risk Sentiment Uncertainty Index (RSUI), and the World Uncertainty Index (WUI). Standard errors are clustered at the Region×Year level. Significance levels are denoted by: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$.

Table 6: Conditional mean and quantile regressions for GDP growth (Temperature levels)

	Full Sample				Poor				Non-poor			
Model	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Panel A: Conditional mean												
C_{it}	-0.534*	0.037	0.296	-0.460	0.128	-0.971**	-3.765**	-1.794**	-0.750**	0.355	2.293***	0.096
	(0.284)	(0.307)	(0.726)	(0.321)	(0.322)	(0.433)	(1.458)	(0.774)	(0.325)	(0.328)	(0.781)	(0.340)
$C_{it} \times C_{gt}$			-0.013				0.141**				-0.101***	
			(0.033)				(0.065)				(0.035)	
Panel B: Quantile τ=.05												
C_{it}	2.336***	1.186**	-0.913	0.175	2.826***	-1.409*	-4.629*	-2.677**	2.288***	1.942***	1.439	1.148*
	(0.591)	(0.569)	(1.237)	(0.617)	(0.646)	(0.781)	(2.390)	(1.245)	(0.687)	(0.643)	(1.281)	(0.661)
$C_{it} \times C_{gt}$			0.109**				0.165				0.024	
			(0.055)				(0.104)				(0.059)	
Panel C: Quantile τ=.25												
C_{it}	0.289	0.425	-0.114	-0.227	0.900**	-1.125**	-4.071**	-2.140**	0.118	0.905**	1.995**	0.501
	(0.347)	(0.374)	(0.804)	(0.386)	(0.379)	(0.499)	(1.623)	(0.872)	(0.399)	(0.411)	(0.845)	(0.410)
$C_{it} \times C_{gt}$			0.028				0.150**				-0.058	
			(0.035)				(0.070)				(0.037)	
Panel D: Quantile τ=.5												
C_{it}	-0.587**	0.029	0.305	-0.460	0.041	-0.967**	-3.753***	-1.784**	-0.787***	0.358	2.295***	0.106
	(0.265)	(0.300)	(0.718)	(0.313)	(0.308)	(0.427)	(1.438)	(0.755)	(0.301)	(0.320)	(0.772)	(0.326)
$C_{it} \times C_{gt}$			-0.014				0.141**				-0.102***	
			(0.032)				(0.064)				(0.035)	
Panel E: Quantile τ=.75												
C_{it}	-1.459***	-0.363	0.723	-0.672**	-0.754**	-0.810*	-3.445**	-1.464*	-1.702***	-0.194	2.597***	-0.240
	(0.224)	(0.272)	(0.784)	(0.326)	(0.297)	(0.462)	(1.533)	(0.795)	(0.254)	(0.285)	(0.866)	(0.349)
$C_{it} \times C_{gt}$			-0.057				0.133*				-0.146***	
			(0.037)				(0.071)				(0.041)	
Panel F: Quantile τ=.95												
C_{it}	-3.045***	-1.013***	1.419	-1.023**	-2.255***	-0.554	-2.927	-0.976	-3.386***	-1.080***	3.083**	-0.779
	(0.293)	(0.344)	(1.133)	(0.480)	(0.424)	(0.683)	(2.157)	(1.074)	(0.350)	(0.377)	(1.253)	(0.519)
$C_{it} \times C_{gt}$			-0.127**				0.119				-0.217***	
			(0.057)				(0.102)				(0.063)	
Observations	9761	9761	9761	3081	3665	3665	3665	1338	6096	6096	6096	1743
Units	194	194	194	92	71	71	71	40	123	123	123	52
Periods	59	59	59	34	59	59	59	34	59	59	59	34
Country FE (1 = Yes)	1	1	1	1	1	1	1	1	1	1	1	1
Region×Year FE (1 = Yes)	0	1	1	1	0	1	1	1	0	1	1	1

Notes: The table reports estimated coefficients from panel quantile regressions with GDP growth as the dependent variable. C_{it} denotes the country-level temperature shock, corresponding to the average temperature itself. C_{gt} is the global average temperature. Specification (1) includes only country fixed effects; specification (2) adds both country and Region×Year fixed effects; specification (3) augments (2) with an interaction term between C_{it} and C_{gt}; and specification (4) further controls for the Financial Stress Index (FSI), the Risk Sentiment Uncertainty Index (RSUI), and the World Uncertainty Index (WUI). Standard errors are clustered at the Region×Year level. Significance levels are denoted by: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$.

Table 7: Conditional mean and quantile regressions for the growth rate of real domestic absorption (Baseline)

	Full Sample				Poor				Non-poor			
Model	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Panel A: Conditional mean												
C_{it}	0.189	0.463	0.352	0.204	-0.734	-1.935**	-2.008*	-1.973	0.499	1.187***	1.002**	1.052**
	(0.415)	(0.414)	(0.456)	(0.568)	(0.644)	(0.907)	(1.056)	(1.902)	(0.467)	(0.427)	(0.463)	(0.518)
$C_{it} \times C_{gt}$			-1.583				-3.052				-2.598	
			(2.310)				(5.120)				(2.238)	
Panel B: Quantile τ=.05												
C_{it}	1.438	1.512**	1.707**	0.657	0.168	-0.876	-0.103	-0.405	1.868*	2.403***	2.310***	1.589
	(0.919)	(0.766)	(0.820)	(1.265)	(1.361)	(1.899)	(2.219)	(4.301)	(1.012)	(0.809)	(0.845)	(1.107)
$C_{it} \times C_{gt}$			-2.019				-4.997				-2.768	
			(3.984)				(10.202)				(3.836)	
Panel C: Quantile τ=.25												
C_{it}	0.588	0.827	0.818	0.383	-0.437	-1.546	-1.297	-1.382	0.935	1.635***	1.490***	1.272*
	(0.547)	(0.508)	(0.548)	(0.793)	(0.816)	(1.182)	(1.387)	(2.655)	(0.607)	(0.542)	(0.573)	(0.715)
$C_{it} \times C_{gt}$			-1.733				-3.779				-2.661	
			(2.698)				(6.474)				(2.602)	
Panel D: Quantile τ=.5												
C_{it}	0.154	0.455	0.339	0.201	-0.753	-1.943**	-2.013*	-1.951	0.450	1.154***	0.986**	1.058**
	(0.398)	(0.407)	(0.446)	(0.555)	(0.628)	(0.892)	(1.039)	(1.888)	(0.443)	(0.412)	(0.450)	(0.511)
$C_{it} \times C_{gt}$			-1.578				-3.047				-2.596	
			(2.275)				(5.053)				(2.204)	
Panel D: Quantile τ=.75												
C_{it}	-0.250	0.091	-0.118	0.045	-1.049*	-2.357***	-2.746***	-2.570	0.006	0.734**	0.520	0.869**
	(0.328)	(0.367)	(0.412)	(0.495)	(0.600)	(0.859)	(0.990)	(1.596)	(0.363)	(0.357)	(0.406)	(0.443)
$C_{it} \times C_{gt}$			-1.431				-2.298				-2.535	
			(2.195)				(4.987)				(2.260)	
Panel F: Quantile τ=.95												
C_{it}	-0.915**	-0.445	-0.814	-0.222	-1.586*	-2.948**	-3.804**	-3.546	-0.678	0.143	-0.130	0.614
	(0.436)	(0.441)	(0.498)	(0.766)	(0.916)	(1.289)	(1.483)	(2.606)	(0.454)	(0.402)	(0.476)	(0.581)
$C_{it} \times C_{gt}$			-1.207				-1.219				-2.450	
			(2.714)				(7.261)				(2.948)	
Observations	9258	9258	8257	3059	3599	3599	3156	1341	5659	5659	5101	1718
Units	176	176	176	91	65	65	65	40	111	111	111	51
Periods	59	59	50	34	59	59	50	34	59	59	50	34
Country FE (1 = Yes)	1	1	1	1	1	1	1	1	1	1	1	1
Region×Year FE (1 = Yes)	0	1	1	1	0	1	1	1	0	1	1	1

Notes: The table reports estimated coefficients from panel quantile regressions with the growth rate of real domestic absorption (real consumption plus investment) as the dependent variable. C_{it} denotes the country-level temperature shock, constructed as the residuals from an AR(2) model of temperature deviations relative to the preceding 30-year moving average. C_{gt} is defined analogously using the global average temperature. Specification (1) includes only country fixed effects; specification (2) adds both country and Region×Year fixed effects; specification (3) augments (2) with an interaction term between C_{it} and C_{gt}; and specification (4) further controls for the Financial Stress Index (FSI), the Risk Sentiment Uncertainty Index (RSUI), and the World Uncertainty Index (WUI). Standard errors are clustered at the Region×Year level. Significance levels are denoted by: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$.

Table 8: Conditional mean and quantile regressions for the growth rate of welfare relevant total factor productivity (Baseline)

	Full Sample				**Poor**				**Non-poor**			
Model	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
					Panel A: Conditional mean							
C_{it}	-0.465	-0.131	-0.387	-0.202	-2.326***	-2.358*	-2.746*	-4.396*	-0.117	0.342	0.072	0.370
	(0.339)	(0.346)	(0.390)	(0.583)	(0.887)	(1.375)	(1.475)	(2.574)	(0.324)	(0.345)	(0.398)	(0.512)
$C_{it} \times C_{gt}$			-0.416				0.201				-1.593	
			(1.660)				(4.516)				(1.776)	
					Panel B: Quantile τ=.05							
C_{it}	0.502	0.184	0.201	-0.307	-2.028	-3.477	-3.257	-5.948	0.911*	1.064*	1.016	0.830
	(0.596)	(0.651)	(0.718)	(1.180)	(1.932)	(2.773)	(3.049)	(6.211)	(0.538)	(0.575)	(0.662)	(0.895)
$C_{it} \times C_{gt}$			2.900				8.220				1.015	
			(3.165)				(8.268)				(3.179)	
					Panel C: Quantile τ=.25							
C_{it}	-0.142	-0.016	-0.174	-0.243	-2.234**	-2.878	-2.980	-4.987	0.232	0.623	0.417	0.554
	(0.396)	(0.425)	(0.474)	(0.775)	(1.073)	(1.938)	(2.101)	(3.685)	(0.370)	(0.403)	(0.458)	(0.637)
$C_{it} \times C_{gt}$			0.785				3.877				-0.640	
			(1.948)				(5.486)				(2.027)	
					Panel D: Quantile τ=.5							
C_{it}	-0.477	-0.137	-0.394	-0.202	-2.334***	-2.321*	-2.733*	-4.453*	-0.119	0.331	0.062	0.369
	(0.334)	(0.340)	(0.383)	(0.574)	(0.870)	(1.326)	(1.429)	(2.597)	(0.320)	(0.338)	(0.390)	(0.501)
$C_{it} \times C_{gt}$			-0.455				0.001				-1.620	
			(1.638)				(4.454)				(1.750)	
					Panel D: Quantile τ=.75							
C_{it}	-0.812**	-0.249	-0.605*	-0.164	-2.422**	-1.843	-2.511**	-3.861*	-0.497	0.060	-0.281	0.210
	(0.330)	(0.332)	(0.367)	(0.490)	(0.941)	(1.137)	(1.225)	(2.223)	(0.322)	(0.336)	(0.387)	(0.433)
$C_{it} \times C_{gt}$			-1.650				-3.475				-2.568	
			(1.835)				(5.455)				(1.953)	
					Panel F: Quantile τ=.95							
C_{it}	-1.364***	-0.414	-0.914*	-0.105	-2.578*	-1.361	-2.293	-2.984	-1.089**	-0.322	-0.759	-0.022
	(0.447)	(0.447)	(0.480)	(0.649)	(1.477)	(1.412)	(1.562)	(3.631)	(0.430)	(0.426)	(0.483)	(0.466)
$C_{it} \times C_{gt}$			-3.388				-6.906				-3.888	
			(2.743)				(7.578)				(2.792)	
Observations	5915	5915	5294	2488	1737	1737	1592	808	4178	4178	3702	1680
Units	117	117	117	74	35	35	35	24	82	82	82	50
Periods	59	59	50	34	59	59	50	34	59	59	50	34
Country FE (1 = Yes)	1	1	1	1	1	1	1	1	1	1	1	1
Region×Year FE (1 = Yes)	0	1	1	1	0	1	1	1	0	1	1	1

Notes: The table reports estimated coefficients from panel quantile regressions with the growth rate of welfare relevant total factor productivity as the dependent variable. C_{it} denotes the country-level temperature shock, constructed as the residuals from an AR(2) model of temperature deviations relative to the preceding 30-year moving average. C_{gt} is defined analogously using the global average temperature. Specification (1) includes only country fixed effects; specification (2) adds both country and Region×Year fixed effects; specification (3) augments (2) with an interaction term between C_{it} and C_{gt}; and specification (4) further controls for the Financial Stress Index (FSI), the Risk Sentiment Uncertainty Index (RSUI), and the World Uncertainty Index (WUI). Standard errors are clustered at the Region×Year level. Significance levels are denoted by: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$.

inflation are negative and statistically significant in the non-poor subsample with estimates fluctuating between -19.9% under specification (2) and -18.8% under specification (3). The effects across the conditional distribution are asymmetric, with positive and non-significant values at quantile $\tau = 0.05$ and negative and significant values fro $\tau = 0.25$ and above. Also, the magnitude of the effects in absolute values increase with the quantile level. For poor countries, the effects are no significant although magnitudes are negative and also increasing with the quantile level. The same sign evidence between responses of GDP growth and inflation talk of temperature shocks as demand-type shocks, aligned to the evidence in Ciccarelli and Marotta (2024) for the conditional average.

Table 9: Conditional mean and quantile regressions for CPI inflation (Baseline)

	Full Sample				**Poor**				**Non-poor**			
Model	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
	Panel A: Conditional mean											
C_{it}	-5.477	-14.483**	-18.082**	-16.815*	-0.352	-2.194	-2.523	-5.989	-5.671	-14.878**	-18.845**	-17.590*
	(4.314)	(7.050)	(8.517)	(8.808)	(1.244)	(3.307)	(3.422)	(4.660)	(4.478)	(7.251)	(8.884)	(9.395)
$C_{it} \times C_{gt}$			-77.388				-9.170				-80.946	
			(52.068)				(13.426)				(53.901)	
	Panel B: Quantile τ=.05											
C_{it}	0.440	9.310	21.020	18.352	2.507*	-1.021	-0.677	4.548	0.200	15.375	21.143	40.575
	(2.309)	(9.045)	(14.828)	(15.678)	(1.462)	(3.677)	(4.445)	(36.422)	(2.214)	(11.939)	(15.998)	(28.372)
$C_{it} \times C_{gt}$			89.802				-15.048				90.761	
			(87.859)				(26.115)				(88.654)	
	Panel C: Quantile τ=.25											
C_{it}	-2.196**	-7.959**	-9.755**	-10.321*	1.178	-1.282	-1.390	-2.264	-2.325**	-8.239**	-10.467**	-10.468
	(1.046)	(3.816)	(4.290)	(5.745)	(1.084)	(3.441)	(3.682)	(5.499)	(1.089)	(4.069)	(4.763)	(6.459)
$C_{it} \times C_{gt}$			-41.787*				-12.777				-44.972*	
			(23.690)				(18.445)				(26.463)	
	Panel D: Quantile τ=.5											
C_{it}	-3.665	-13.447**	-16.812**	-16.243**	-0.125	-2.145	-2.332	-4.754	-3.712	-13.872**	-18.809**	-16.138**
	(2.351)	(6.127)	(7.268)	(7.968)	(1.151)	(3.211)	(3.301)	(4.710)	(2.333)	(6.309)	(8.112)	(8.140)
$C_{it} \times C_{gt}$			-71.959				-9.778				-80.793	
			(45.591)				(12.984)				(51.056)	
	Panel D: Quantile τ=.75											
C_{it}	-7.923	-31.173**	-38.529**	-31.701**	-1.441	-3.128	-3.689	-10.167**	-8.131	-33.581**	-43.441**	-31.918**
	(6.760)	(15.111)	(18.723)	(15.754)	(1.563)	(4.005)	(4.252)	(4.743)	(6.992)	(15.967)	(20.700)	(15.132)
$C_{it} \times C_{gt}$			-164.816				-5.460				-186.559	
			(119.498)				(21.769)				(131.474)	
	Panel F: Quantile τ=.95											
C_{it}	-21.211	-78.801**	-103.759*	-75.123*	-4.340	-4.435	-4.910	-14.460	-22.499	-85.262**	-110.153*	-77.916*
	(20.827)	(39.921)	(54.646)	(39.611)	(2.956)	(6.006)	(5.950)	(14.736)	(22.490)	(42.337)	(56.388)	(41.778)
$C_{it} \times C_{gt}$			-443.726				-1.572				-473.019	
			(340.143)				(36.139)				(347.497)	
Observations	2671	2671	2415	1341	308	308	293	176	2363	2363	2122	1165
Units	61	61	61	42	11	11	11	7	50	50	50	35
Periods	60	60	50	34	60	60	50	34	60	60	50	34
Country FE (1 = Yes)	1	1	1	1	1	1	1	1	1	1	1	1
Region×Year FE (1 = Yes)	0	1	1	1	0	1	1	1	0	1	1	1

Notes: The table reports estimated coefficients from panel quantile regressions with CPI inflation as the dependent variable. C_{it} denotes the country-level temperature shock, constructed as the residuals from an AR(2) model of temperature deviations relative to the preceding 30-year moving average. C_{gt} is defined analogously using the global average temperature. Specification (1) includes only country fixed effects; specification (2) adds both country and Region×Year fixed effects; specification (3) augments (2) with an interaction term between C_{it} and C_{gt}; and specification (4) further controls for the Financial Stress Index (FSI), the Risk Sentiment Uncertainty Index (RSUI), and the World Uncertainty Index (WUI). Standard errors are clustered at the Region×Year level. Significance levels are denoted by: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$.

4.6 Limitations

Our application provides valuable insights into the effects of temperature shocks on the conditional distribution of macroeconomic outcomes, but the results should be interpreted with caution given several limitations. First, the analysis implicitly treats the average temperature as a sufficient statistic for climatic conditions. However, climate change is a multidimensional phenomenon that also involves changes in the variability and distribution of temperatures, the frequency of extremes, and other meteorological characteristics. A natural extension would be to incorporate indicators of temperature or precipitation extremes, as in Chauvet et al. (2025) and Kim et al. (2025), or to include a richer set of climate predictors following the approach of Akyapı et al. (2025). Second, the current framework is static and does not model dynamics explicitly. The literature on dynamic panel quantile regressions remains limited, with Galvao (2011) providing one of the few contributions that address the Nickell (1981) bias problem known from the least-squares context when the time dimension is modest. To the best of our knowledge, an extension of the MMQREG framework that accommodates lagged dependent variables or dynamic feedback mechanisms has not yet been developed. Third, the conditional location–scale specification, while flexible, imposes a parametric form that may restrict the heterogeneity of responses across units. In particular, the coefficient associated with the climate shock is assumed to be homogeneous across countries, which may mask meaningful cross-sectional variation in climate sensitivity. Furthermore, the model implicitly assumes that the determinants of conditional mean growth and of tail risk are the same, up to the location–scale structure discussed above. Economically, one might expect additional drivers for downside risk (institutional quality, financial conditions, etc.) that need not matter for the mean in the same way. Future research could explore more flexible specifications allowing for heterogeneous slopes, random-coefficient structures, or hierarchical extensions of the MMQREG estimator to capture such differences systematically.

5 Conclusions

This paper extended the quantile–via–moments (MMQREG) estimator of Machado and Santos Silva (2019) to settings with multiple high-dimensional fixed effects and alternative variance-covariance estimators. The proposed framework enables researchers to estimate quantile regression models in data sets characterized by complex hierarchical structures and unobserved heterogeneity, while retaining the computational simplicity of the original approach. Such an extension is particularly valuable in applications where group-specific effects influence not only the location but also the dispersion of the conditional distribution of the outcome.

Through a series of Monte Carlo experiments, we demonstrated that the proposed estimator accurately identifies the parameters of interest even in the presence of two sets of fixed effects. The bias-corrected version based on the split-panel jackknife of Dhaene and Jochmans (2015) effectively mitigates small-sample bias, though at the cost of somewhat higher sampling variability. Our simulations further highlighted the critical role of inference procedures: while feasible GLS standard errors perform well under correctly specified heteroskedasticity, they become unstable when the predicted scale component approaches zero. In contrast, heteroskedasticity-robust and clustered standard errors deliver more reliable coverage, particularly under intra-cluster correlation, with the latter offering the best finite-sample performance in large panels.

We illustrated the empirical relevance of the methodology in a cross-country Climate Growth-at-Risk application. Using the proposed estimator, we studied how temperature shocks affect the conditional distribution of GDP growth, accounting for multiple sources of unobserved heterogeneity through country and Region×Year fixed effects. The results revealed that temperature shocks increase downside risks to output in poorer economies. These findings underscore the value of modeling heterogeneity along the entire conditional distribution rather than focusing solely on mean effects, as they uncover climate-induced vulnerabilities that standard approaches overlook.

The extended MMQREG framework broadens the empirical scope of quantile regression analysis. It offers a tractable and theoretically grounded approach to modeling heterogeneous effects in complex panel structures, enabling richer distributional analyses in fields such as climate economics, macro-finance, and development policy. Nonetheless, several avenues for future research remain open. An important extension would be to incorporate dynamics into the MMQREG framework to account for state dependence and persistence in the conditional distribution of outcomes. Further progress could also be made by allowing for heterogeneous slope coefficients, random-coefficient specifications, or hierarchical structures that capture systematic differences across units or groups. Developing these extensions would expand the applicability of the MMQREG approach to a wider range of empirical settings while deepening our understanding of heterogeneity in economic relationships.

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A Derivation of the influence functions

A.1 Model Identification

The MMQREG approach assumes that the DGP is linear in parameters, with an heteroskedastic error term that is also a linear function of parameters:

$$y_i = x_i'\beta + \nu_i,$$
$$\nu_i = \varepsilon_i \times x_i'\gamma,$$

where ε is an unobserved i.i.d. random variable that is independent of x and such that $x\gamma$ is larger than 0 for any x. In this case, the τ-th conditional quantile model can be written as:

$$Q_y(\tau|X) = x_i'(\beta + Q_\varepsilon(\tau) \times \gamma)$$

.

This model is identified under the following conditions:

$$
\begin{aligned}
E[(y_i - x_i'\beta)x_i] &= E[h_{1,i}] = 0 \\
E[(|y_i - x_i'\beta| - x_i'\gamma)x_i] &= E[h_{2,i}] = 0 \\
E\left[1\left(Q_\varepsilon(\tau)x_i'\gamma + x_i'\beta \geq y_i\right) - \tau\right] &= E[h_{3,i}] = 0
\end{aligned}
$$

The rest of the appendix uses q_τ to represent $Q_\varepsilon(\tau)$.

A.2 Estimation of the Variance-Covariance Matrix

To estimate the variance-covariance matrix of the set of coefficients $\theta' = (\beta',\ \gamma',\ q_\tau)$, we need to obtain the influence functions of all coefficients, which are defined as:

$$
\lambda_i = \bar{G}(\theta)^{-1} \begin{bmatrix} h_{1,i} \\ h_{2,i} \\ h_{3,i} \end{bmatrix}
$$

where the Jacobian matrix $\bar{G}(\theta)$ is defined as

$$
\bar{G}(\theta) = \begin{bmatrix} \bar{G}_{11} & \bar{G}_{12} & \bar{G}_{13} \\ \bar{G}_{21} & \bar{G}_{22} & \bar{G}_{23} \\ \bar{G}_{31} & \bar{G}_{32} & \bar{G}_{33} \end{bmatrix}
$$

with

$$
\bar{G}_{jk} = -\frac{1}{N}\sum_{i=1}^{N} \frac{\partial h_{j,i}}{\partial \theta_k'} \ \forall j, k \in 1, 2, 3
$$

First Moment Condition. Lets consider the expression inside the expectation of the first moment condition:

$$
h_{1,i} = x_i(y_i - x_i'\beta)
$$

that identifies the location model. Simple algebra is used to show that:

$$
\begin{aligned}
\bar{G}_{11} &= -\frac{1}{N}\sum_{i=1}^{N} \frac{\partial h_{1,i}}{\partial \beta'} \\
&= -\frac{1}{N}\sum_{i=1}^{N}(-x_i x_i') \\
&= N^{-1}X'X
\end{aligned}
$$

$$
\bar{G}_{12} = \bar{G}_{13} = 0
$$

Second Moment Condition. Now, lets consider the expression inside the expectation of the second moment condition

$$
h_{2,i} = x_i(|y_i - x_i'\beta| - x_i'\gamma)
$$

that identifies the scale parameter. It is possible to show that:

$$\bar{G}_{21} = -\frac{1}{N}\sum \frac{\partial h_{2,i}}{\partial \beta'}$$
$$= \frac{1}{N}\sum x_i x_i' \frac{y_i - x_i'\beta}{|y_i - x_i'\beta|}$$

where $\frac{y_i - x_i'\beta}{|y_i - x_i'\beta|} = sign(y_i - x_i'\beta)$. Under the assumption $\varepsilon_i \times x_i\gamma$, or in this case $y_i - x_i'\beta$, is uncorrelated with x_i, we can simplify the expression as:

$$\bar{G}_{21} = N^{-1}\left(N^{-1}\sum sign(y_i - x_i'\beta)\right)\sum x_i x_i'$$
$$= N^{-1}E[sign(y_i - x_i'\beta)]X'X.$$

Additionally,

$$\bar{G}_{22} = -\frac{1}{N}\sum \frac{\partial h_{2,i}}{\partial \gamma'}$$
$$= \frac{1}{N}\sum x_i x_i'$$
$$= N^{-1}X'X.$$

$$\bar{G}_{23} = 0$$

.

Third Moment Condition. Regarding the expression inside the expectation of the third moment condition

$$h_{3,i} = 1\left(q_\tau x_i'\gamma + x_i'\beta - y_i \geq 0\right) - \tau \text{ or}$$
$$h_{3,i} = 1\left(q_\tau \geq \frac{y_i - x_i'\beta}{x_i'\gamma}\right) - \tau = 1\big(q_\tau \geq \varepsilon\big) - \tau$$

,

that is used to identify the quantile of standardized residual, notice that the indicator function 1() is not differentiable. We borrow from the nonparametric literature to approximate this function with a kernel. Call $k()$ a well behaved kernel function that is symmetric around 0, and $K()$ its integral, with range between 0 and 1. With an arbitrarily small bandwidth h, we can use the function $K()$ to approximate the indicator function as:

$$\lim_{h\to 0} K\left(\frac{z}{h}\right) \approx 1(z \geq 0)$$

Thus the function $h_{3,i}$ can be approximated as

$$h_{3,i} \approx K\left(\frac{1}{h}\Big(q_\tau x_i'\gamma + x_i'\beta - y_i\Big)\right) - \tau$$

The Jacobian matrix $\bar{G}_{31}$ can be then obtained as:

$$\begin{aligned}
\bar{G}_{31} &= -\frac{1}{N}\sum \frac{\partial h_{3,i}}{\partial \beta'} \\
&= -N^{-1}\sum \frac{1}{h} k\left(\frac{1}{h}\Big(q_\tau x_i'\gamma + x_i'\beta - y_i\Big)\right) x_i' \\
&= -N^{-1}\sum \frac{1}{h} k\left(\frac{1}{h}(q_\tau x_i'\gamma - \nu_i)\right) x_i' \\
&= -N^{-1}\sum \frac{1}{h} k\left(\frac{1}{h}(q_\tau - \varepsilon_i) x_i'\gamma\right) x_i'.
\end{aligned}$$

If we condition on X, use the law of iterated expectations, and assume that ε_i is homoskedastic, we obtain:

$$\begin{aligned}
\bar{G}_{31} &= -N^{-1}\sum \frac{1}{h} k\left(\frac{1}{h}(q_\tau - \varepsilon_i) x_i'\gamma\right) x_i' \\
&= -N^{-1}\sum \frac{1}{h} k\left(\frac{q_\tau - \varepsilon_i}{h}\right) \frac{x_i'}{x_i'\gamma} \\
&= -N^{-1} f_\varepsilon(q_\tau)\sum \frac{x_i'}{x_i'\gamma}
\end{aligned}$$

Finally, this simplifies to:

$$\bar{G}_{31} \simeq -f_\varepsilon(q_\tau)\frac{\bar{x}_i'}{\bar{x}_i'\gamma},$$

where we use the fact that asymptotically, the expression $\frac{1}{N}\sum \frac{a_i}{b_i}$ can be approximated using Taylor expansions by $\frac{\bar{a}}{\bar{b}}$. $\bar{G}_{32}$ can be derived similarly:

$$\begin{aligned}
\bar{G}_{32} &= -\frac{1}{N}\sum \frac{\partial h_{3,i}}{\partial \gamma'} \\
&= -N^{-1}\sum \frac{1}{h} k\left(\frac{1}{h}(q_\tau - \varepsilon_i) x_i'\gamma\right) q_\tau x_i' \\
&= -N^{-1}\sum \frac{1}{h} k\left(\frac{q_\tau - \varepsilon_i}{h}\right) q_\tau \frac{x_i'}{x_i'\gamma} \\
&\simeq -f(q_\tau) q_\tau \frac{\bar{x}'}{\bar{x}'\gamma}
\end{aligned}$$

while $\bar{G}_{3,3}$ is given by:

$$\begin{aligned}
\bar{G}_{3,3} &= -\frac{1}{N}\sum \frac{\partial h_{3,i}}{\partial q_\tau} \\
&= -N^{-1}\sum \frac{1}{h} k\left(\frac{1}{h}(q_\tau - \varepsilon_i) x_i'\gamma\right) x_i'\gamma \\
&= -N^{-1}\sum \frac{1}{h} k\left(\frac{q_\tau - \varepsilon_i}{h}\right) \frac{x_i'\gamma}{x_i'\gamma} \\
&\simeq -f(q_\tau)
\end{aligned}$$

A.3 Influence Functions

Location Coefficients: To estimate the influence functions of the location coefficients, notice that:

$$\lambda_i(\beta) = \bar{G}_{11}^{-1} h_{1,i} = N(X'X)^{-1}(x_i(y_i - x_i'\beta)) = N(X'X)^{-1}(x_i \nu_i)$$

which can also be written as a function of the standardized residuals:

$$\lambda_i(\beta) = \bar{G}_{11}^{-1} h_{1,i} = N(X'X)^{-1}(x_i(y_i - x_i'\beta)) = N(X'X)^{-1}(x_i(x_i'\gamma \times \varepsilon)).$$

Scale Coefficients: Similarly, for the scale coefficients,

$$\begin{aligned}
\lambda_i(\gamma) &= \bar{G}_{22}^{-1}\Big(h_{2,i} - \bar{G}_{2,1}\lambda_i(\beta)\Big) \\
&= N(X'X)^{-1}\Big(x_i(|\nu_i| - x_i'\gamma) - N^{-1}E[sign(\nu_i)]X'X\big[N(X'X)^{-1}(x_i\nu_i)\big]\Big) \\
&= N(X'X)^{-1}\Big(x_i(|\nu_i| - x_i'\gamma) - E[sign(\nu_i)](x_i\nu_i)\Big) \\
&= N(X'X)^{-1}x_i\Big(|\nu_i| - E[sign(\nu_i)]\nu_i - x_i'\gamma\Big)
\end{aligned}$$

However,

$$\begin{aligned}
|\nu_i| &= \nu_i \times 1(\nu_i \geq 0) - \nu_i \times 1(\nu_i < 0) \\
|\nu_i| &= \nu_i \times 1(\nu_i \geq 0) - \nu_i \times [1 - 1(\nu_i \geq 0)] \\
|\nu_i| &= 2\nu_i \times 1(\nu_i \geq 0) - \nu_i
\end{aligned}$$

and

$$\begin{aligned}
E[sign(\nu_i)] &= E[1(\nu_i \geq 0)] - E[1(\nu_i < 0)] \\
E[sign(\nu_i)] &= E[1(\nu_i \geq 0)] - E[(1 - 1(\nu_i \geq 0))] \\
E[sign(\nu_i)] &= 2E[1(\nu_i \geq 0)] - 1
\end{aligned}$$

Thus,

$$\begin{aligned}
\lambda_i(\gamma) &= N(X'X)^{-1}x_i\Big(2\nu_i \times 1(\nu_i \geq 0) - \nu_i - (2E[1(\nu_i \geq 0)] - 1)\nu_i - x_i'\gamma\Big) \\
&= N(X'X)^{-1}x_i\Big(2\nu_i \times 1(\nu_i \geq 0) - 2E[1(\nu_i \geq 0)]\nu_i - x_i'\gamma\Big) \\
&= N(X'X)^{-1}x_i\Big(2\nu_i \times \big[1(\nu_i \geq 0) - E[1(\nu_i \geq 0)]\big] - x_i'\gamma\Big) \\
&= N(X'X)^{-1}x_i\Big(\tilde{\nu}_i - x_i'\gamma\Big)
\end{aligned}$$

This last expression is the equivalent simplification used in Machado and Santos Silva (2019) and Im (2000). If the scale function is strictly positive, it also follows that $1(\nu_i \geq 0) = 1(\varepsilon_i \geq 0)$. Thus, it can be simplified as

$$\lambda_i(\gamma) = N(X'X)^{-1}x_i(x_i'\gamma) \times (\tilde{\varepsilon}_i - 1).$$

Quantile of Standardized Residual: Finally, for the quantile of standardized residual, observe that:

$$\begin{aligned}
\lambda_i(q_\tau) &= \bar{G}_{33}^{-1}\Big(h_{3,i} - \bar{G}_{31}\lambda_i(\beta) - \bar{G}_{32}\lambda_i(\gamma)\Big) \\
&= -\frac{1}{f_\varepsilon(q_\tau)} \times \Bigg(\Big(1(q_\tau \geq \varepsilon) - \tau\Big) \\
&+ f_\varepsilon(q_\tau)\frac{\bar{x}_i'}{\bar{x}_i'\gamma}N(X'X)^{-1}x_i(x_i'\gamma \times \varepsilon) \\
&+ f_\varepsilon(q_\tau)q_\tau\frac{\bar{x}_i'}{\bar{x}_i'\gamma}N(X'X)^{-1}x_i\big(\tilde{\nu}_i - x_i'\gamma\big)\Bigg) \\
&= \frac{\tau - 1\big(q_\tau \geq \varepsilon\big)}{f_\varepsilon(q_\tau)} - \frac{x_i'\gamma \times \varepsilon_i}{\bar{x}_i'\gamma} - q_\tau\frac{\tilde{\nu}_i - x_i'\gamma}{\bar{x}_i'\gamma}
\end{aligned}$$